Combinatorial Identities

One of my fascinations is with combinatorial Identities. Especially the ones I happen upon while doing other things. Here are some of the ones that I have discovered. Many such identities and general methods for deriving identities amongst combinatorial coefficients can be found in the beautiful little book by Graham et all, [1, Chapter 5].

1.  
\[ \sum_{r=0}^{n} \binom{2n}{n+r} = \frac{n}{2} \binom{2n}{n} \]

2.  
\[ \sum_{r=0}^{n} \binom{2n}{r} = \frac{1}{2} \binom{2n}{n} + 2^{2n-1} \]

3.  
\[ \sum_{r=0}^{n} \frac{2n+1}{2n+1-r} \binom{2n}{r} = 2^{2n} \]

4.  
\[ \sum_{r=0}^{n-\alpha} \frac{2n+1-2r}{2n+1-r} \binom{2n}{r} = \binom{2n}{n+\alpha} = \binom{2n}{n-\alpha} \]

5.  
\[ \sum_{k=0}^{n} \binom{2k}{k} \binom{2(n-k)}{n-k} = 2^{2n} \]

References