CSCI 2400 – Models of Computation

Solution for Homework #2

1. Give regular expressions for the following languages on \( \Sigma = \{a, b, c\} \).

   (a) all strings containing exactly one a.

   \textit{Solution}
   \[(b + c)^*a(b + c)^*\]

   (b) all strings containing no more than three a’s.

   \textit{Solution}
   \[(b + c)^*(a + \lambda)(b + c)^*(a + \lambda)(b + c)^*(a + \lambda)(b + c)^*\]

   (d) all strings that contain no run of a’s of length greater than two.

   \textit{Solution}
   \[(b + c)^* + (b + c)^*((a + aa)(b + c)^+)^*(a + aa)(b + c)^*\]

   (e) all strings in which all runs of a’s have lengths that are multiples of three.

   \textit{Solution}
   \[(b + c)^*((aaa)(b + c)^*)^*\]

2. Find a regular grammar that generates the language \( L(aa^* (ab + a)^*) \).

   \textit{Solution}
   \[G = (V, T, S, P), \text{ where}\]
   \[V = \{S, A, B\}, \]
   \[T = \{a, b\}, \]
   \[P = \{S \to aA, \ A \to aA|aB|\lambda, \ B \to bA\}\]

   The derivation of a string \( aaaababa \):
   
   \[S \Rightarrow aA \Rightarrow aaA \Rightarrow aaAa \Rightarrow aaAaaB \Rightarrow aaaabA \Rightarrow aaaabaB \]
   \[\Rightarrow aaaababA \Rightarrow aaaababaA \Rightarrow aaaababa.\]
3. Find a regular grammar that generates the language on $\Sigma = \{a, b\}$ consisting of all strings with no more than three a’s.

Solution
$G = (V, T, S, P)$, where
$V = \{S, A, B\}$,
$T = \{a, b\}$,
$P = \{S \rightarrow bS|aA|\lambda, A \rightarrow bA|aB|\lambda, B \rightarrow bB|aC|\lambda, C \rightarrow bC|\lambda\}$

The derivation of a string $babaaab$:
$S \Rightarrow bS \Rightarrow baA \Rightarrow bbaA \Rightarrow bbaabA \Rightarrow babbaB$
$\Rightarrow babbaaC \Rightarrow babaabC \Rightarrow babbaab.$

4. Find regular grammar for the following languages on $\{a, b\}$.
$L=\{w : (n_a(w) - n_b(w)) \mod 3 = 1\}$

Solution
$G = (V, T, S, P)$, where
$V = \{S, A, B\}$,
$T = \{a, b\}$,
$P = \{S \rightarrow aA|bB, A \rightarrow aB|bS|\lambda, B \rightarrow aS|bA\}$

The derivation of a string $abaaaba$:
$S \Rightarrow aA \Rightarrow abS \Rightarrow abaA \Rightarrow abaaB \Rightarrow abaaS$
$\Rightarrow abaaA \Rightarrow abaaaabS \Rightarrow abaaaabaA \Rightarrow abaaaba.$

![Finite Automaton Diagram](image)

Figure 1: finite automaton accepting $L=\{w : (n_a(w) - n_b(w)) \mod 3 = 1\}$
5. The \textit{min} of a language $L$ is defined as

$$\text{min}(L) = \{ w \in L : \text{there is no } u \in L, v \in \Sigma^+, \text{ such that } w = uv \}.$$ 

\textit{Solution}

Take the transition graph of a DFA for $L$ and delete all edges going out of any final vertex. Note that this works only if we start with a DFA!