Fuzzy Inference Systems

(Chapter 4)

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The Big Picture

Inference systems consist of three components:
• rule base
• data base (defines membership functions)
• reasoning mechanism (aggregation)
• defuzzification

Motivation

Fuzzy Reasoning left us with something similar to:

\[ \begin{array}{c}
\text{C} \\
\text{z is C} \\
\text{Z} \\
\end{array} \]

Real world:
Sensor measurements are crisp (albeit imprecise)
Output to controller has to be crisp

Outline

Introduction/Motivation
Mamdani Model
TSK Model
Tsukamoto Model
et cetera

Fuzzy Inference System

Fuzzy Inference System

a.k.a.:
- fuzzy rule based system
- fuzzy expert system
- fuzzy model
- fuzzy associative memory
- fuzzy logic controller
- fuzzy system
Design of IS

Consistency:
no rules with the same antecedents but different consequents

Completeness:
for any $x \in X$
there is at least one rule $j$ s.t. $\mu_{i,j} \neq 0$

Defuzzification Requirements

- Intuition:
  A crisp value should represent the fuzzy set from an intuitive point of view (e.g., max. membership grade)
- Computational Burden:
  simple (real-time constraints)
- Continuity: small changes in fuzzy sets should not result in large changes of $z$

Defuzzification: COA

Center of Area

\[ z_{\text{COA}} = \frac{\int_{\mu_{A}} \mu_{A}(z)dz}{\int_{\mu_{A}} \mu_{A}(z)dz} \]

Defuzzification: BOA

Bisector of area:

\[ z_{\text{BOA}} = \frac{\int_{\alpha}^{\beta} \mu_{A}(z)dz}{\int_{\mu_{A}} \mu_{A}(z)dz} \]

where

$\alpha = \min \{ z : \mu_{A}(z) \neq 0 \}$
$\beta = \max \{ z : \mu_{A}(z) \neq 0 \}$

Defuzzification: MOM

Mean of maximum:

\[ z_{\text{MOM}} = \frac{\int_{\mu_{A}} z \mu_{A}(z)dz}{\int_{\mu_{A}} \mu_{A}(z)dz} \]

\[ Z = \{ z : \mu_{A}(z) = \max \mu_{A}(z) \} \]
**Fuzzy Rules and Fuzzy Reasoning**

**Center Average Defuzzifier**

Approximation of COA Defuzzifier by average of center of areas of fuzzy sets s.t.

\[ z_{CA} = \frac{\sum_{j=1}^{n} z_j w_j}{\sum_{j=1}^{n} w_j} \]

where

- \( z_j \): center of area of \( j \)th fuzzy set \( B \)
- \( w_j \): height of \( j \)th fuzzy set \( B \)

**SISO: max-min composition centroid defuzzification**

If \( X \) is small then \( Y \) is small
If \( X \) is medium then \( Y \) is medium
If \( X \) is large then \( Y \) is large

**MISO: max-min composition centroid defuzzification**

If \( X \) is small and \( Y \) is small then \( Z \) is negative large
If \( X \) is small and \( Y \) is large then \( Z \) is negative small
If \( X \) is large and \( Y \) is small then \( Z \) is positive small
If \( X \) is large and \( Y \) is large then \( Z \) is positive large

**Sugeno Model (TSK)**

Combines fuzzy sets in antecedents with crisp function in output:

- IF \( x \) is small AND \( y \) is \( B \) THEN \( z = f(x,y) \)
  - \( f \) does not follow compositional rule of inference

- IF \( x \) is small THEN \( Y = 4 \)
- IF \( X \) is medium THEN \( Y = 0.5X + 4 \)
- IF \( X \) is large THEN \( Y = X - 1 \)

**Sugeno: MISO**

IF \( X \) is small AND \( Y \) is small THEN \( z = x + y + 1 \)
IF \( X \) is small AND \( Y \) is large THEN \( z = y + 3 \)
IF \( X \) is large and \( Y \) is small \( z = x + 3 \)
IF \( X \) is large and \( Y \) is large THEN \( z = x + y + 2 \)

**Tsukamoto Model**

- Consequent of rule is represented by monotonical MF
- Crisp output is induced by firing strength
- Overall output: weighted average of rule output
- BUT: not as transparent
Tsukamoto Example

IF X is small THEN Y is C₁
IF X is medium THEN Y is C₂
IF X is large THEN Y is C₃

Summary

Fuzzy Inference:
- Mamdami
  Defuzzification derived from area via appropriate method
- Sugeno
  functional relation of output
- Tsukamoto