Fuzzy Inference Systems

(Chapter 4)

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Fuzzy Inference System

a.k.a.:
- fuzzy rule based system
- fuzzy expert system
- fuzzy model
- fuzzy associative memory
- fuzzy logic controller
- fuzzy system
The Big Picture

Inference systems consist of three components:
• rule base
• database (defines membership functions)
• reasoning mechanism (aggregation)
• defuzzification

Outline

Introduction/Motivation
Mamdani Model
TSK Model
Tsukamoto Model
et cetera
Motivation

Fuzzy Reasoning left us with something similar to:

Real world:
Sensor measurements are crisp (albeit imprecise)
Output to controller has to be crisp

Fuzzy Inference System

Input $x$ $\rightarrow$ $x$ is $A_1$ $\rightarrow$ $y$ is $B_1$ $\rightarrow$ $w_1$
$\rightarrow$ $x$ is $A_2$ $\rightarrow$ $y$ is $B_2$ $\rightarrow$ $w_2$
$\rightarrow$ $\cdots$
$\rightarrow$ $x$ is $A_r$ $\rightarrow$ $y$ is $B_r$ $\rightarrow$ $w_r$

Aggregator $\rightarrow$ $\text{Defuzzify} \rightarrow y$
Design of IS

Consistency:
- no rules with the same antecedents but different consequents

Completeness:
- for any \( x \in X \)
- there is at least one rule j s.t. \( \mu_{A_j} \neq 0 \)

Mamdani Model

- Coherent modeling of inference system
- Appropriate defuzzification strategy
Defuzzification Requirements

- Intuition:
  A crisp value should represent the fuzzy set from an intuitive point of view (e.g., max. membership grade)

- Computational Burden:
  simple (real-time constraints)

- Continuity: small changes in fuzzy sets should not result in large changes of $z$

Defuzzification: COA

Center of Area

$z_{COA} = \frac{\int z \mu_A(z) dz}{\int \mu_A(z) dz}$

+ intuitive
+ smooth
- comp. burden
Defuzzification: BOA

Bisector of area:

\[
\int_{\alpha}^{\beta} \mu_{A}(z)dz = \int_{z_{BOA}}^{z_{BOA}} \mu_{A}(z)dz
\]

\[
\alpha = \min\{z|z \in Z\}
\]

\[
\beta = \max\{z|z \in Z\}
\]

Defuzzification: MOM

Mean of maximum:

\[
Z' = \{z|\mu_{A}(z) = \max_{z} \mu_{A}(z)\}
\]
**Fuzzy Rules and Fuzzy Reasoning**

### Center Average Defuzzifier

Approximation of COA Defuzzifier by average of center of areas of fuzzy sets s.t.

\[
z_{CA} = \frac{\sum_{j=1}^{n} z_j w_j}{\sum_{j=1}^{n} w_j}
\]

where

- \(z_j\) center of area of \(j^{th}\) fuzzy set \(B\)
- \(w_j\) height of \(j^{th}\) fuzzy set \(B\)

### SISO: max-min composition centroid defuzzification

If X is small then Y is small
If X is medium then Y is medium
If X is large then Y is large
**MISO: max-min composition centroid defuzzification**

If X is small and Y is small then Z is negative large
If X is small and Y is large the Z is negative small
If X is large and Y is small the Z is positive small
If X is large and Y is large then Z is positive large

**Sugeno Model (TSK)**

Combines fuzzy sets in antecedents with crisp function in output:

IF x is A AND y is B THEN z=f(x,y)

(Does not follow compositional rule of inference)

IF x is small THEN Y=4
IF X is medium THEN Y=-0.5X+4
IF X is large THEN Y=X-1
Sugeno: MISO

IF X is small AND Y is small THEN z = -x + y + 1
IF X is small AND Y is large THEN z = -y + 3
IF X is large and Y is small THEN z = -x + 3
IF X is large and Y is large THEN z = x + y + 2

Tsukamoto Model

- Consequent of rule is represented by monotonical MF
- Crisp output is induced by firing strength
- Overall output: weighted average of rule output
- BUT: not as transparent
**Fuzzy Rules and Fuzzy Reasoning**

**Tsukamoto Example**

IF X is small THEN Y is C\(_1\)

IF X is medium THEN Y is C\(_2\)

IF X is large THEN Y is C\(_3\)

![Membership Grades](image)

(a) Antecedent MFs

(b) Consequent MFs

(c) Each Rule's Output

(d) Overall Input-Output Curve

**Summary**

Fuzzy Inference:

- Mamdami
  
  *Defuzzification derived from area via appropriate method*

- Sugeno
  
  *Functional relation of output*

- Tsukamoto
last slide