Homework 8

Return by: Thursday, November 30

1. Consider the traveling salesperson problem:
   We have a map with $n$ cities $c_1, c_2, \ldots, c_n$. For every pair of cities $c_i, c_j$ there is a road that connects the two cities and this road has length $d_{ij}$. The various road lengths may be different. The problem is to find the shortest route, starting from city $c_1$, that goes through all the cities, and returns back to city $c_1$. This is the route that the salesperson will follow.

   (a) Describe an algorithm for a nondeterministic Turing machine that finds the shortest route in polynomial number of time steps (with respect to $n$). (Each time step corresponds to a transition of the machine.)

   (b) Describe an algorithm for a deterministic Turing machine that finds the shortest route. Is the time of your algorithm polynomial?

2. (a) Give the algorithm of a Turing Machine that enumerates the strings of the following language in proper order.

   $$L = \{a^n b^n : n \geq 1\}$$

   (b) For this language, is the proper order the same with the alphabetical order? Explain your answer.

3. (a) Prove that the union of two countable sets is a countable set.

   (b) Use the result of (a) to prove that the set of non recursively enumerable languages is not countable.

4. Show that if a language is not recursively enumerable then its complement cannot be recursive.

5. Suppose that language $L$ is such that there is a Turing machine that enumerates the elements of $L$ in proper order. Show that this means that $L$ is recursive.