CSCI 2400 – Models of Computation, Section 3

Homework 9

Return by: Friday, December 8

1. Prove that the following problem is undecidable. Given any Turing machine $M$, a symbol $a$, and string $w$, determine whether or not the symbol $a$ is ever written on the tape when $M$ is applied to input $w$. (Hint: reduce the halting problem to this problem.)

2. Prove that the following problem is undecidable. Determine whether or not an arbitrary Turing machine halts on all input. (Hint: reduce the halting problem to this problem.)

3. Let $b(n)$ be the maximum number of head moves made by any $n$-state Turing machine that halts when started with a blank tape. Show that $b(n)$ is not computable. (Hint: look at Example 12.3, page 319.)

4. Show that the following problem is undecidable. For two recursively enumerable languages $L_1$ and $L_2$, determine whether $L_1 \subseteq L_2$. (Hint: use the result of Theorem 12.3, page 322.)

5. Let $A = 001, 0011, 11, 101$ and $B = 01, 111, 111, 010$.
   (a) Does the pair $A, B$ have a PC-solution?
   (b) Does it have an MPC-solution?