1. (10 points) Using a truth table, show that the sentence \( \neg(P \lor Q) \iff \neg P \land \neg Q \) is valid. (See section 13.5.4 in the text for a definition of valid.) Note that this sentence is one of de Morgan’s laws.

2. (16 points) Transform the following sentences into Conjunctive Normal Form and simplify them as much as possible.
   
   (a) \((P \lor (Q \land R)) \Rightarrow X\)
   
   (b) \(\neg(A \Rightarrow (B \lor C)) \land D\)
   
   (c) \((\neg P \land R) \lor (S \Rightarrow \neg T)\)
   
   (d) \((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)\)

3. (20 points) Translate the following problem statements into Propositional logic:

   On Saturdays in the summer, it is either sunny or rainy but not both. If it is sunny, then Alice goes on a picnic with her friends and plays softball. If it is rainy, they go to the gym and play basketball. If Alice plays basketball or softball, then she gets exercise.

   Use the following predicates:

   - \(S\) = sunny
   - \(R\) = rainy
   - \(P\) = Alice goes on a picnic
   - \(G\) = Alice goes to the gym
   - \(F\) = Alice plays softball
   - \(B\) = Alice plays basketball
   - \(E\) = Alice gets exercise

   Show that Alice gets exercise by doing a refutation proof using resolution.

4. (16 points) Do exercise 13.3 in the text.

5. (20 points) Using the predicates \(\text{Likes}(x, y)\) (i.e. \(x\) likes \(y\)) and \(\text{Hates}(x, y)\) (i.e. \(x\) hates \(y\)), translate the following English sentences into first order logic.

   (a) Alice likes everyone that likes Bob.
   
   (b) Bob likes everyone that Alice likes.
   
   (c) There is someone who likes everyone that Alice hates.
   
   (d) No one likes anyone that Alice hates.
   
   (e) Not everyone hates the people that like Alice.

   For example “Everyone who likes Bob likes Alice” becomes \(\forall_x \text{Likes}(x, Bob) \Rightarrow \text{Likes}(x, Alice)\).

6. (16 points) Transform the following sentences into conjunctive normal form (use Universal elimination, Existential elimination, and And-elimination as necessary):

   (a) \(\forall_x A(x) \Rightarrow \forall_y B(x, y) \Rightarrow \neg C(y)\)
   
   (b) \(\neg \forall_x A(x) \Rightarrow \neg B(x)\)
   
   (c) \(\neg \exists_x A(x) \land B(x) \land \neg C(x)\)
   
   (d) \(\forall_y (\exists_y A(x, y) \land \neg B(y)) \Rightarrow D(x)\)

   For example \(\forall_x A(x) \land B(x) \Rightarrow C(x) \lor D(x)\) becomes \(\neg A(x) \lor \neg B(x) \lor C(x) \lor D(x)\).