Linearity of Expectation

Expectation of a sum is sum of expectations.
Examples.
Expected value of a product?
1. $E[X] = \sum_\omega P(\omega)X(\omega) = \sum_k k \, P \{ X = k \}$.

2. Examples: dice sum; uniform distribution; Bernoulli; Binomial; Expected number of trials to success (success probability $p$).

3. Conditional Expectation:

$$E[X|A] = \sum_k k \, P \{ X = k | A \} = \frac{1}{P[A]} \sum k \, P \{ X = k \cap A \}.$$

4. Law of Total Expectation: $E[X] = E[X|A] \, P \{ A \} + E[X|\bar{A}] \, P \{ \bar{A} \}$.

   Expected number of tries to success via law of total expectation: $E[T] = \frac{1}{p}$.
1. **Expected Value of a Sum is the Sum of the Expectations:**

\[ \mathbb{E} \left[ \sum_i a_i X_i \right] = \sum_i a_i \mathbb{E} [X_i]. \]

2. Examples: Dice sum; Binomial; Coupon collector; Random walk.

3. Expected value of a product is the product of expected values?
   
   Example: let \( R_1, R_2 \) be rolls of independent dice. \( \mathbb{E}[R_1^2] \neq \mathbb{E}[R_1]^2; \mathbb{E}[R_1 R_2] = \mathbb{E}[R_1] \mathbb{E}[R_2]. \)

4. Is \( \mathbb{E}[1/X] = 1/ \mathbb{E} [X]? \)

5. Example to introduce variance: expected value is not all there is to the game.