MIDTERM: 90 Minutes

Last Name: ___________________________
First Name: _________________________
RIN: ________________________________
Section: _____________________________

Answer ALL questions. You may use a single sided $8\frac{1}{2} \times 11$ crib sheet.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

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Circle at most one answer per question. 10 points for each correct answer and -5 points for each incorrect answer (blank answer is 0 points). Don’t guess!

(a) \( p \to (q \land r) \) is equivalent to what other compound proposition:

A \((p \to q) \land r\)

B \((p \to q) \land (p \to r)\)

C \((p \land q) \to r\)

D \(p \lor (q \land r)\)

(b) The negation of “If Malik is in pajamas then all lights are off” is

A Malik is in pajamas and at least one light is on

B Malik is in pajamas or all lights are off

C Malik is not in pajamas and at least one light is on

D Malik is not in pajamas and all lights are off

(c) \( P(n) \) is a predicate \((n \) is an integer). \( P(2) \) is true; and, \( P(n) \to (P(n^2) \land P(n - 2)) \) is true for \( n \geq 2 \). For which \( n \) can we be sure \( P(n) \) is true?

A All \( n \geq 2 \).

B All even \( n \geq 0 \).

C All odd \( n \geq 0 \).

D All \( n \) which are perfect squares.

(d) Compute the remainder when \( 2014^{2014} \) is divided by 5? [Hint: \( 2014 \equiv -1 \pmod{5} \).]

A \( r = 1 \)

B \( r = 2 \)

C \( r = 3 \)

D \( r = 7 \)

(e) A friendship network has 7 people and each person has 5 friends. How many edges (friendship links) are there in this friendship network?

A 17 edges

B 18 edges

C Not enough information to determine the number of edges

D This friendship network cannot possibly exist
2. Induction Proofs

1. Prove by induction that for all integer $n \geq 1$: \[ \sum_{i=1}^{n} \frac{1}{i(i+1)} = 1 - \frac{1}{n+1}. \]

2. Suppose $a \equiv b \pmod{k}$. Prove by induction that for all integer $n \geq 1$: $a^n \equiv b^n \pmod{k}$. 
\[
(x \equiv y \pmod{z} \text{ means } x - y \text{ is divisible by } z.)
\]
3 Well formed arithmetic expressions

Define a set \( \mathcal{A} \) of well formed arithmetic strings (sequences) with alphabet \( \Sigma = \{1, +, \times, (, )\} \).

[Base Case] \( 1 \in \mathcal{A} \);

[Recursive Rules]

1. \( x, y, z \in \mathcal{A} \rightarrow (x + y + z) \in \mathcal{A} \)
2. \( x, y \in \mathcal{A} \rightarrow (x \times y) \in \mathcal{A} \).

(a) Of the following three strings, circle the one that is in \( \mathcal{A} \).

\( (1 + 1 + 1) \times (1 + 1) \) \( (1 + 1 + 1) \times ((1 + 1 + 1) + 1 + 1) \) \( ((1 + 1 + 1) \times ((1 + 1 + 1) + 1 + 1)) \)

(b) Give a derivation of the string in (a) that is in \( \mathcal{A} \). (A derivation is a sequence of strings in \( \mathcal{A} \) where each string is obtained from the previous strings by applying one of the recursive rules.)
The function $\text{eval}$ takes an input string from the set $\mathcal{A}$ of well formed arithmetic expressions ($\mathcal{A}$ was defined in problem 3) and computes its value as an arithmetic expression. For example,

$$\text{eval} : \left( (1 + 1 + 1) \times \left( ((1 + 1 + 1) \times (1 + 1 + 1)) + 1 + 1 \right) \right) \mapsto 33$$

**Prove** that for every string $x \in \mathcal{A}$, $\text{eval}(x)$ is **odd**.

Is $\left( (1 + 1 + 1) \times \left( ((1 + 1 + 1) + 1 + 1 + 1) \right) \right)$ in $\mathcal{A}$? If yes, give a **derivation**. If no, **prove** it.
5 Rooted binary trees (NOT rooted full binary trees)

Give the recursive definition of rooted binary trees. Explicitly state your base case and recursive rules.

Let $F$ be the number of full nodes (with 2 children) and $L$ the number of leaf nodes (with no children). For any non-empty rooted binary tree, prove that

$$L = F + 1.$$
SCRATCH