QUIZ 3: 90 Minutes

Last Name: ______________________
First Name: ______________________
RIN: ______________________
Section: ______________________

Answer ALL questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

| Total | 100 |
1. The first 2 questions refer to the following experiment.

There are two identical bags. One contains 3 white and 1 black ball; the other 1 white and 3 black balls. You pick a bag randomly (probability \( \frac{1}{2} \) for each bag) and then randomly pick one of the balls in the bag (probability \( \frac{1}{4} \) for each ball). You got a white ball. Let \( X \) be the number of white balls in the other bag. (The information that you got a white ball is very important.)

What is \( E[X] \) (expected value)?

A 1
B \( \frac{6}{4} \)
C \( \frac{10}{4} \)
D 2
E \( \frac{5}{4} \)

2. What is \( Var(X) \) (variance)?

A \( \frac{2}{4} \)
B \( \frac{3}{4} \)
C 1
D \( \frac{5}{4} \)
E \( \frac{6}{4} \)

3. A game costs \( x \) to play. You toss 4 fair coins. If you get more heads than tails, you win and get back \$10 + x \) for a profit of \$10. Otherwise, you lose and get nothing back, so your loss is \$x. What is an expression for your expected profit in dollars?

A \( 10 \times \frac{1}{2} - x \times \frac{1}{2} \)
B \( \frac{50 - 11x}{16} \)
C \( \frac{60 - 10x}{16} \)
D \( \frac{50 - x}{16} \)
E \( \frac{60 - x}{16} \)
4. A Martian couple continues to have children until they have 2 males (not necessarily in a row). On Mars, males are twice as likely as females. Assume children are independent. Let $X$ be the number of children this couple will have. What is $\mathbb{E}[X]$, the expected number of children this couple will have?

A 2
B 3
C 2.5
D 3.5
E 4

5. You toss 5 independent fair coins. What is the probability that you will get 4 or more heads?

A \(\binom{5}{4} \times \frac{1}{2^5}\)
B \(\frac{3}{16}\)
C \(\frac{5}{32}\)
D \(\frac{1}{4}\)
E \(\frac{9}{32}\)

6. Step 1: Toss 9 fair coins. Step 2: if you got more heads than tails in Step 1, toss 9 more coins and stop; if you get fewer heads than tails in Step 1, stop. Let $X$ be the number of heads you toss. What is $\mathbb{E}[X]$?

A 6.25
B 6.75
C 7.25
D 9
E 8
7. Language $\mathcal{L}_1 = \{\text{all non-empty strings in which the number of 1's is even}\}$. Which finite automaton solves this problem, i.e. the YES-set (set of accepted strings) for the automaton is $\mathcal{L}_1$?

8. Language $\mathcal{L}_2 = \{\text{all strings in which the number of 1's is even}\}$ which CFG solves this problem - i.e., generates the strings in $\mathcal{L}_2$?

9. Which of the following is countable?

10. Which of the following is not a valid way to show that a set $S$ is countable: