Financial Markets: Very Noisy Information Processing

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In the definition of continuous compatibility (definition A.1 in the appendix) it is not made clear how close data sets would be defined for data sets with different numbers of data points or different input values. For example if $\mathcal{D}'$ is simply $\mathcal{D}$ with one of the points in $\mathcal{D}$ repeated, then the two data sets contain exactly the same information, and should thus be considered close. In fact proposition B.4 uses a definition of continuous compatibility defined with respect to a more general notion of “close data sets”. The following notation applies to definition A.1

Let $\mathcal{S}$ be the compact support for $dF(x)$. Let $\mathcal{D} = \{x_i, y_i\}_{i=1}^N$, $\mathcal{D}' = \{x_i', y_i'\}_{i=1}^{N'}$ be any two data sets on $\mathcal{S}$ such that for every $\{x_i, y_i\}$ in $\mathcal{D}$ there is an $\{x_i', y_i'\}$ in $\mathcal{D}'$ such that $|x_i - x_j| \leq \varepsilon_{\text{max}}$ and $|y_i - y_j'| \leq \varepsilon_{\text{max}}$ and the same is true for every $\{x_j', y_j'\}$ in $\mathcal{D}'$, i.e. there is an $\{x_i, y_i\}$ in $\mathcal{D}$ such that $|x_i - x_j| \leq \varepsilon_{\text{max}}$ and $|y_i - y_j'| \leq \varepsilon_{\text{max}}$. Let $\mathcal{A}(\mathcal{D}) = g(x)$, $\mathcal{A}(\mathcal{D}') = g(x) + \eta(x)$.

**Definition A.1** $\mathcal{L}$ is $n^{th}$ order-continuously compatible if $\exists C$ such that

$$\langle |\eta(x)|^n \rangle_x \leq (C\varepsilon_{\text{max}})^n$$

with probability 1 (i.e. for almost every $\mathcal{D}$). We will write $\mathcal{L} \in \mathcal{CC}_n$.

Note that all that has been updated is the notion of what close data sets are.