8.1 Assume \( TSP(G, b) \) if \( \text{returns} \) \( \text{false} \) if no tour of length \( b \) or less exists \( G \).

Sum up all the distances of \( G \).

\( TSP_{opt} \) calls \( TSP(G, b) \) as a binary search primitive.

\( TSP_{opt}(G) \)

let \( S \) be the sum of all distances.

return \( \text{Binary Search}(G, 0, S) \)

\( \text{Binary Search Tour}(G, l, u) \)

\[ b = l + u/2 \]

if \( TSP(G, b) \neq \text{false} \)

return \( \text{Binary Search Tour}(G, l, b) \)

else return \( \text{Binary Search Tour}(G, b, u) \)
8.4

a. Clique 3 can be checked in polynomial time.

b. Reduction is in the wrong reduction.

   We must reduce known NP-complete Problem Clique to Clique 3.

c. C is a vertex if \( V - C \) is an independent set in \( G \).

d. Largest clique can be of size 4.

   Take all possible subsets \( \geq 4 \), to test whether there is clique of size \( k \).
8. 10

a. We can view this as a generalization of the clique problem. Let \( G, k \) be a clique instance.

Construct a subgraph \( H \) which is a clique of size \( k \).

b. This is a generalization of the Ruderka path.

Given a graph with \( n \) vertices, let \( g = n - 1 \).
9.4. Keep inserting the smallest degree vertex in the independent set. Delete that vertex. Repeat the step till no more vertex is left.

St: Largest independent set size is $x$.

For each vertex we would have picked one of the $d+1$ vertices. Size will be at most $\frac{x}{d+1}$. 

9.6. Find the MST of the designated nodes.

Let that cost be $X$.

If the optimal Steiner tree has cost $Y$,

we can do an eulerian traversal of the optimal Steiner

and keeping only the designated vertices.

\[ 2Y \geq X \]

\[ \therefore X \leq 2Y \]
7.1 The optimal solution is on the upper right corner of the convex feasible region (5, 2) and has the value \(5x + 3y = 31\).

7.2 Let \(MN\) be the quantity between Mexico and New York, \(MC\) between Mexico and California, \(KN\) between Kansas and New York, and \(KC\) between Kansas and California.

\[
\begin{align*}
\text{min} & \quad 4MN + MC + 2KN + 2KC \\
MN + KN & = 10 \quad MN + KN \leq 10 \quad MN + KC \geq 10 \\
MC + KC & = 8 \quad MC + KC \leq 13 \quad MC + KC \geq 15 \\
MN + MC & = 6 \quad MN + MC \leq 8 \quad MN + MC \geq 8 \\
KN + KC & = 15 \quad KN + KC \leq 15 \quad KN + KC \geq 15 \\
\end{align*}
\]
\(MN, MC, KN, KC > 0\)

7.3 Let \(q_i\) be the quantity in cubic meters of material \(i\).

\[
\begin{align*}
\text{max} & \quad 1000q_1 + 1200q_2 + 1200q_3 \\
2q_1 + q_2 + 3q_3 & \leq 100 \\
q_1 + q_2 + q_3 & \leq 60 \\
q_1 & \leq 40 \quad q_2 \leq 30 \quad q_3 \leq 20 \\
q_1, q_2, q_3 & > 0
\end{align*}
\]
7.4. Let $R$ be regular CPU bear
$S$ be Strong CPU bear
\[ \max R + 1.5S \]
\[ S \leq 2R \]
\[ R + S \leq 3000 \]
\[ R, S \geq 0 \]

7.10

\[
\begin{align*}
SA & \rightarrow DG T & 4 \\
SA & \rightarrow B E GT & 2 \\
SB & \rightarrow EG T & 1 \\
SC & \rightarrow FT & 4 \\
SC & \rightarrow BE GT & \frac{2}{13}
\end{align*}
\]