A solution to the hat problem
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April 10, 2001

The players generate a strategy as follows. A \emph{configuration} is a particular assignment of red or blue to each player’s hat. Independently and with probability $\ln n/n$, each configuration is designated \emph{special}; other configurations are called \emph{ordinary}. After the hats are drawn each player can narrow the number of possible configurations to two: the actual one and the one where his own hat is the opposite color. If one of those configurations is special and the other is ordinary, the player writes down the ordinary one as his guess; otherwise she makes no guess.

The players win in the following situation: the actual configuration is ordinary and at least one of the configurations at Hamming distance 1 is special. The probability of the former is $1 - \ln n/n$ and the probability of the latter is $1 - (1 - \ln n/n)^n > 1 - e^{-\ln n} = 1 - 1/n$. Hence the probability of winning is $1 - O(\ln n/n)$. 