CSCI 2400 – Models of Computation

Practice Midterm Exam 2

1. Give the NPDA for the following context-free language, describe what each state does:

\[ \{a^n b^k : n > k \geq 0\} \]

2. (a) Give a context-free grammar for the following language:

\[ L = \{wa^nb^nw^R : w \in \{a, b\}^*, n \geq 0\} \]

where \( w \) is any string over the alphabet \( \Sigma = \{a, b\} \) including \( \lambda \).

(b) Give the derivation of the string \( abaabba \) using your grammar.

3. Is the following grammar ambiguous? Prove your answer.

\[
\begin{align*}
S & \rightarrow BA \mid C \\
B & \rightarrow Bb \mid b \\
A & \rightarrow aA \mid a \\
C & \rightarrow bCa \mid ba
\end{align*}
\]

4. Give the Chomsky normal form of the following grammar.

\[
\begin{align*}
S & \rightarrow AbBa \\
A & \rightarrow ABa \mid a \\
B & \rightarrow BaA \mid b
\end{align*}
\]

5. Prove that the following language is not context-free.

\[ L = \{a^i b^j c^k : 0 \leq i \leq j \leq k\} \]

6. Prove that the following language is context-free.

\[ L = \{ww^R : w \neq abba, \text{ and } w \in \{a, b\}^*\} \]