1. (a) Give a recursive algorithm (pseudocode) that computes the sum of the first \( n \) odd positive integers.
   
   (b) Give a proof by induction that your algorithm is correct.

2. Consider the alphabet \( \Sigma = \{A, B, C, D, E, F, G\} \). For each question below, give your answer and a short justification.
   
   (a) How many different strings of length 5 can be generated from the letters in \( \Sigma \)?
   
   (b) How many different strings of length 7 can be generated from the letters in \( \Sigma \) that begin or end with a vowel?
   
   (c) How many different strings of length 4 can be generated from the letters in \( \Sigma \) that contain a single G and a single A, with G preceding A?
   
   (d) How many different strings of length 4 can be generated from the letters in \( \Sigma \) in which no letter is repeated?

3. The RPI Discrete Math Lovers club has 37 members, 20 of whom are women and 17 of whom are men. The club needs to form a 4 person party-planning committee. For each of the committee selection policies below, give your answer and an explanation of how you derived it.
   
   (a) How many total committees are possible?
   
   (b) How many committees are possible if the committee must have 2 women and 2 men?
   
   (c) How many committees are possible if the committee must have at least 1 man?
   
   (d) How many committees are possible if the committee must consist of all women or of all men?

4. Find the number of subsets of the set \( S = \{1, 2, 3, \ldots, 100\} \) that contain exactly 4 elements, the sum of which is even. Give your solution and a short explanation of how you derived it.

5. The figure below shows a 4 by 5 block grid of streets.

   ![Grid Diagram]

   You are at point A and you need to get to point B by traversing the streets (edges) in the grid.
   
   (a) Suppose at each intersection, you can only go up or right. (You are not allowed to go left or down). For example, one allowable route is: Right, Right, Up, Right, Up, Right, Up, Right, Up.
   
   How many different routes are there from A to B? Justify your answer.
   
   (b) Suppose at each intersection, you can go any direction available on the grid. How many different routes are there from A to B? Justify your answer.