1. Recall the definition of the sequence of Fibonacci numbers:

\[
\begin{align*}
  f_0 &= 0 \\
  f_1 &= 1 \\
  f_n &= f_{n-1} + f_{n-2} \quad \text{for } n \geq 2
\end{align*}
\]

Use strong induction to prove that \( f_n \geq \left(\frac{3}{2}\right)^{n-2} \) for \( n \geq 1 \).

2. (a) Give a recursive algorithm (pseudocode) for computing the sum of the first \( n \) positive integers.
   (b) Give an inductive proof that your algorithm is correct.

3. (a) Give a recursive algorithm for computing \( 3^{2^n} \), where \( n \) is a nonnegative integer.
   (b) Give an inductive proof that your algorithm is correct.

4. A DNA sequence is a sequence of letters, each of which is one of A, C, G, or T.
   How many 5-element DNA sequences
   (a) end with A?
   (b) start with T and end with G?
   (c) contain only A and T?
   (d) do not end with C?

5. How many functions are there from the set \( A = \{1, 2, \ldots, n\} \), where \( n \) is a positive integer, to the set \( B = \{0, 1\} \)
   (a) that are injective?
   (b) that assign 0 to both 1 and \( n \)?
   (c) that assign 1 to exactly one of the positive integers less than \( n \)?