Announcements

• Homework 7 is due next Monday, April 20 at 10am in class.

• You have until Friday April 24, 2015 to submit an exam 2 grade appeal (in-person meeting).
  • See last lecture’s slides for appeal policy changes.
Last Time: Laplace’s Theory of Probability

• An **experiment** is a procedure that yields one of a given set of possible **outcomes**.
• The **sample space** of the experiment is the set of possible outcomes.
• An **event** is a subset of the sample space.

• If $S$ is a **finite** sample space of **equally likely** outcomes, and $E$ is an event, that is, $E \subseteq S$, then the **probability** of $E$ is: $p(E) = |E|/|S|$
Example 1

• Suppose you flip a fair coin 3 times. What is the probability that it comes up heads at least twice?
Example

• What is the probability that a 5 card poker hand does not contain the queen of hearts?
Last Time: Probability Distributions

• Let $S$ be the sample space of an experiment with a finite or countable number of outcomes.
• We can assign a probability $p(s)$ to each outcome $s \in S$.
• Require 2 conditions:
  • 1. $0 \leq p(s) \leq 1$ for each $s \in S$
  • 2. $\sum_{s \in S} p(s) = 1$
• $p$ is a function
  • Maps from $S$ to $[0,1]$
• $p$ is called a probability distribution
Example: Assigning Probabilities

• Suppose, when a loaded die is rolled, rolling a 3 is twice as likely as rolling any other number.
  What is the probability distribution?
Uniform Distribution

- **Definition:** Suppose that $S$ is a set with $n$ elements. The uniform distribution assigns the probability $1/n$ to each element of $S$.

- Examples:
  - Probability distribution of flipping a fair coin.
  - Probability distribution of rolling a fair die.
Probability of an Event

• The probability of the event $E$ is the sum of the probabilities of the outcomes in $E$.

\[ p(E) = \sum_{e \in E} p(e) \]
Example: Probability of an Event

• Suppose that a die is biased so that 3 appears twice as often as each other number, but that the other five outcomes are equally likely.
• What is the probability that an odd number appears when we roll this die?
Probabilities of Complements and Unions of Events

• Complements: \( p(\overline{E}) = 1 - p(E) \) still holds.

• Since each outcome is in either \( E \) or \( \overline{E} \), but not both,

\[
\sum_{s \in S} p(s) = 1 = p(E) + p(\overline{E}).
\]

• Unions: \( p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2) \)

also still holds for probability distributions.
Conditional Probability

• A bit string of length 4 is generated at random. What is the probability that it contains at least 2 consecutive 0s given that the first bit is a 0?

• Let $E$ and $F$ be events with $p(F) > 0$. The conditional probability of $E$ given $F$, denoted by $P(E|F)$, is defined as:

$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$
Illustration of Conditional Probability

\[ p(E|F) = \frac{p(E \cap F)}{p(F)} \]
Conditional Probability

\[ p(E|F) = \frac{p(E \cap F)}{p(F)} \]

- A bit string of length 4 is generated at random. What is the probability that it contains at least 2 consecutive 0s given that the first bit is a 0?
More Conditional Probability

\[ p(E|F) = \frac{p(E \cap F)}{p(F)} \]

- What is the conditional probability that a family with two children has two boys, given that they have at least one boy. Assume that each of the possibilities \( BB, BG, GB, \) and \( GG \) is equally likely, where \( B \) represents a boy and \( G \) represents a girl.
Water Balloon Russian Roulette
Russian Roulette

• We are playing water balloon Russian roulette with a 6-chamber revolver. I put two bullets in adjacent chambers, spin, point the gun at my head, and pull the trigger. Click. I'm still dry. It's now your turn. I hand the gun to you and give you two choices.
  • a) Re-spin, aim at your own head and pull the trigger.
  • b) Do not spin, aim at your own head, and pull the trigger.

• Which do you choose, assuming you want to stay dry?
• Strategy (a): spin
• Strategy (b): don’t spin
Independence

• **Definition**: The events $E$ and $F$ are independent if and only if $p(E \cap F) = p(E)p(F)$
The Birthday Problem

• What is the minimum number of people who need to be in a room so that the probability that at least 2 share the same birthday is $\frac{1}{2}$?

  • Assume
    • Birthdays are independent events.
    • Each birthday is equally likely.
Good Problems to Review

- Rosen 7.2: 5, 7, 9, 11, 13, 15, 17, 19, 21, 25, 27, 31