Announcements

• Homework 4 will be posted on the web site today.
  • It is due Thursday, March 12 at 10am in class.

• Homework 3 was returned in recitation yesterday.
  • You can appeal grades until next Wednesday.
Exam Grades

• The average exam grade was 63 (the median was also 63).
• I do not curve individual exams.
  • I will curve the final course grades if needed.
• I also do not think 63 is the right average for an exam grade.
  • I would prefer 70-71.
• So, if I curved individual exams, I would curve this one.
  • With many disclaimers, it would add about 8 points to each grade.
  • * This does not mean I am curving this exam.
Exam Grade Appeals

• The exam solutions are posted online.
  • The name of who graded each problem is listed with the solution.
• Exam grade appeals can be made until next Friday 3/13.
  • First read the solutions.
  • Then, See the grader of the problem you are concerned about in office hours when possible, or by appointment if necessary.
• Finally, if you are not satisfied with the outcome of the TA meeting, see me (in office hours when possible or by appointment when necessary).
Last Time Functions
Last Time Functions
• Section 2.3 question 32
• Section 2.3 question 40

• Question about whether you can find bijection onto all integers.
• Section 2.2 35

• Last year homework 4 question 1.
SEQUENCES AND SUMMATION

Section 2.4
Sequences

• Imagine a person (with a lot of spare time) who decides to count her ancestors.
• She has two parents, four grandparents, eight grand-grandparents, etc.
• We can write this in a table

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
</tr>
</tbody>
</table>

• Can guess that the $k^{th}$ element is $2^k$.
  • Just a guess – we would need to prove this.
Sequences

- A sequence is a ordered list of elements.
  - Each element has a unique position in the list.
- Formally, a sequence is a function from a subset of the integers to a set $S$.
  - Usually maps from the set $\{0, 1, 2, 3, 4, \ldots\}$ or $\{1, 2, 3, 4, \ldots\}$ to the set $S$.

- We do not write $f(n)$ for an element in a sequence.
- Instead, the notation $a_n$ is used to denote the image of the integer $n$.
  - The sequence is $\{a_0, a_1, a_2, a_3, \ldots\}$
- We call $a_n$ a term of the sequence.
Example of Sequence

\[ a_n = \frac{1}{n} \quad \{a_n\} = \{a_1, a_2, a_3, \ldots \} \]
Formula for a Sequence?

• Can we find an explicit formula for the $n^{th}$ term given only the first few elements of a sequence?

• Examples:
  • 7, 11, 15, 19, 23, 27, 31, 35, ...
  • 3, 6, 11, 18, 27, 38, 51, 66, 83, ...
  • 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, ...
  • O, T, T, F, F, S, S, E, ...

• To do this, try to find a pattern
  • Are terms obtained from previous terms by adding the same amount, or an amount that depends on position in the sequence?
  • Are terms obtained from previous terms by multiplying by a particular amount?
  • Are terms obtained by combining previous terms in a certain way?
Arithmetic Progression

• An arithmetic progression is a sequence of the form:
  \[a, a + d, a + 2d, \ldots, a + nd, \ldots\]
  where the initial term \(a\) and the common difference \(d\) are real numbers.

• Another way to write this is
  \[a + nd, \, n = 0, 1, 2, \ldots\]

Examples:

1. Let \(a = -1\) and \(d = 4\):

2. Let \(a = 7\) and \(d = -3\):
Geometric Progression

• A geometric progression is a sequence of the form:
  
  \[ a, ar, ar^2, \ldots, ar^n, \ldots \]

  where the initial term \( a \) and the common ratio \( r \) are real numbers.

• Another way to write this is \( t_n = ar^n, \ n = 0, 1, 2, \ldots \)

• Examples:
  1. Let \( a = 1 \) and \( r = -1 \).
  2. Let \( a = 2 \) and \( r = 5 \).
  3. Let \( a = 6 \) and \( r = 1/3 \).
Recurrence Relations

• A recurrence relation for the sequence \( \{a_n\} \) is an equation that expresses \( a_n \) in terms of one or more of the previous terms of the sequence.
  - Example: \( \{a_n\} \) is the sequence that satisfies \( a_n = a_{n-1} + 3 \) for \( n = 1, 2, 3, 4, \ldots \).

• The initial conditions for a sequence specify the terms that precede the first term where the recurrence relation takes effect.
  - Example initial conditions: \( a_0 = 2 \)

• What are \( a_1, a_2 \) and \( a_3 \)?
Example Recurrence Relation

• Let \( \{a_n\} \) be a sequence that satisfies the recurrence relation \( a_n = a_{n-1} - a_{n-2} \) for \( n = 2, 3, 4, \ldots \)

• The initial conditions are \( a_0 = 3 \) and \( a_1 = 5 \).

• What are \( a_2 \) and \( a_3 \)?
Fibonacci Sequence

Define the **Fibonacci sequence** $f_0, f_1, f_2, \ldots$, by:

- Initial Conditions: $f_0 = 0$, $f_1 = 1$
- Recurrence Relation: $f_n = f_{n-1} + f_{n-2}$

**Example:** Find $f_2, f_3, f_4$, and $f_5$. 
Solving Recurrence Relations

• Finding a formula for the $n^{th}$ term of the sequence generated by a recurrence relation is called **solving the recurrence relation**.

• Such a formula is called a **closed formula**.

• **Example:**
  • Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for $n = 2,3,4,\ldots$.
  • Is $a_n = 3n$ is a solution?
Solving Recurrence Relations (cont.)

- Let \( \{a_n\} \) be a sequence that satisfies the recurrence relation
  \[ a_n = 2a_{n-1} - a_{n-2} \] for \( n = 2, 3, 4, \ldots \).

- Is \( a_n = 2^n \) a solution?

- Is \( a_n = 5 \) a solution?
Iterative Solution Example

Let \( \{a_n\} \) be a sequence that satisfies the recurrence relation

\[ a_n = a_{n-1} + 3 \text{ for } n = 2, 3, 4, \ldots \]

Suppose that \( a_1 = 2 \).

Finding a Solution - Method 1: forward substitution
Iterative Solution Example

• Let \( \{a_n\} \) be a sequence that satisfies the recurrence relation
  \[ a_n = a_{n-1} + 3 \text{ for } n = 2,3,4,\ldots \]
  Suppose that \( a_1 = 2 \).

• **Find a Solution – Method 2**: backward substitution
Compound Interest Example

• Suppose a person deposits $10,000 in a savings account at a bank yielding 11% per year with interest compounded annually. How much will be in the account after 30 years?
Good Problems to Review

• Section 2.3: 21, 23, 31, 33, 39
• Section 2.4: 1, 3, 9, 11, 13, 15, 17, 19