CSci 2300 — Data Structures and Algorithms
Week 3, Class 6 — SOLUTIONS

Here’s a solution to the section problem from the Class 5 notes:

```cpp
Polynomial operator * ( const Polynomial & f, const Polynomial & g )
{
    Polynomial h;
    int i, j;
    for ( i=0; i<f.terms_.size(); ++i )
        for ( j=0; j<g.terms_.size(); ++j ){
            double coeff = f.terms_[i].coefficient * g.terms_[j].coefficient;
            double expon = f.terms_[i].exponent + g.terms_[j].exponent;
            h.terms_.push_back( PTerm( coeff, expon ) );
        }
    sort( h.terms_.begin(), h.terms_.end() );

    Polynomial h2;
    i=0;
    while ( i<h.terms_.size() ) {
        int expon = h.terms_[i].exponent;
        PTerm pt( 0.0, expon );
        for( ; i < h.terms_.size() && h.terms_[i].exponent == expon; ++i ){
            pt.coefficient += h.terms_[i].coefficient;
        }
        h2.terms_.push_back( pt );
    }

    return h2;
}
```

Here’s the problem from Class 6

1. (10 points) Give a proof by induction that \( \sum_{i=1}^{n} 2i - 1 = n^2 \) for \( n \geq 1 \).

   Solution:

   **Basis case:** For \( n = 1 \), the summation has only a single term:
   \( 2(1) - 1 = 1 \). Since \( 1^2 = 1 \), the basis case is established.
**Induction step**: For $n > 1$, suppose that for all $k$ such that $1 \leq k < n$,

\[
\sum_{i=1}^{k} (2i - 1) = k^2.
\]

Then,

\[
\sum_{i=1}^{n} (2i - 1) = \sum_{i=1}^{n-1} (2i - 1) + 2n - 1
\]

\[
= (n - 1)^2 + (2n - 1) \quad \text{by the Induction Hypothesis}
\]

\[
= n^2 - 2n + 1 + 2n - 1 = n^2.
\]