Exercises

1. (6 points) For each pair of functions, $T(n)$ and $f(n)$, determine which of the following hold:

\[ T(n) = O(f(n)) \quad T(n) = \theta(f(n)) \]

Justify your answers. (Assume $a$ and $b$ are unspecified constants greater than 1 and $a > b$.)

(a) $T(n) = n^2 \log n + 5n$, $f(n) = n^3$

**Solution:** $T(n) = O(f(n))$, but $T(n) \neq \theta(f(n))$. The reason is that the limit of $n^2 \log n/n^3$ is equal to the limit of $\log n/n$, which is 0.

(b) $T(n) = \log_a n$, $f(n) = \log_b n$

**Solution:** $T(n) = O(f(n))$ and $T(n) = \theta(f(n))$. The reason is that $\log_a n = \log_b n/\log_b a$, so the limit of $T(n)/f(n)$ is $1/\log_b a$, which is a constant.

2. (6 points) Give the best possible $O$ estimate for $T(n)$.

(a) $T(n) = (n^3 + 10n^2) \cdot (n^3 \log n + 20n^4)$

**Solution:** The first factor (expression in parentheses) is $O(n^3)$, and the second is $O(n^4)$. Multiplying these gives $O(n^7)$.

(b) $T(n) = n^{3n} + n^{10} + 1500n^3 \log n.$

**Solution:** As discussed in class, $T(n) = O(n^{3n})$. You can’t eliminate factors that involve $n$. The solution $O(n^3)$ is strictly correct, but not as informative.

3. (5 points) Give the best possible “O” estimate for the computation time required by the following code as a function of $n$, the length of the array.

```c++
// assume arr is an array containing n integers
int k = 5;
for ( int i=0; i<=n-k; i++ ) {
    sum = 0;
    for ( int j=i; j<i+k; j++ ) {
        sum += arr[j];
    }
    cout << "Sum of elements " << i << " through " << i+k-1 << " is " << sum << "\n";
}
```
Solution: The inner loop requires $k$ iterations and each iteration requires constant time. But, since $k = 5$, this is $O(1)$, i.e. constant time. Since the outer runs for $n - k$ iterations and there is constant time per iteration, the overall result is $O(n)$.

4. Reading: The rest of Chapter 2. You should understand at the very least, the definition for the Max Subsequence Sum problem. (3 points)
Find the maximum subsequence sum of

\[-1, 6, -3, 2, -7, 6, 8, -3, 4, -2, 1\]

Solution: 15 This is the subsequence 6, 8, -3, 4.