Statistical and Learning Techniques in Computer Vision
Homework 2: Due Thursday September 14, 2006

1. (12 points)
   (a) In class we used the fact that when
   \[ p(x) \propto e^{-\frac{1}{2}(ax^2 - 2bx)}, \]
   then \( p(x) \) is a normal distribution with variance \( \sigma^2 = 1/a \) and mean \( b/a \).
   Prove the vector version of this result:
   \[ p(x) \propto e^{-\frac{1}{2}x^T Ax - 2y^T x} \]
   with symmetric, positive definite matrix \( A \), then \( p(x) \) is multivariate normal.
   In doing so, derive the mean \( \mu \) and covariance \( \Sigma \) of \( x \) in terms of \( A \) and \( y \).
   Recall that the general form of the multivariate normal distribution is
   \[ f(u; \mu, \Sigma) = Ce^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}, \]
   where \( C \) is chosen to ensure that \( f \) integrates to 1.
   (b) Use the foregoing result to derive the posterior distribution of the mean \( \mu \)
       of a multivariate normal distribution given samples \( X = \{x_1, \ldots, x_N\} \),
       known covariance \( \Sigma \), and a multivariate normal prior on \( \mu \) with mean \( \mu_0 \)
       and covariance \( \Sigma_0 \).

2. (8 points) Derive the mean and variance of \( \hat{p}(x) \) using Parzen windows with a
   Gaussian kernel function.

3. (10 points) Prove that the \( k \)-nearest neighbor approximation to the density is
   not differentiable and is not a density. To do this, consider points in only 1
   dimension, let \( k = 2 \), and assume the points are distinct.