Definition 1: a grasp has form closure iff it cannot be moved when the fingers are locked

\[ G_n^T \nu \geq 0 \Rightarrow \nu = 0 \]

Definition 2: a grasp has form closure iff it is possible for frictionless fingers to generate any wrench.

\[ \left\{ \begin{array}{l} G_n \lambda_n = g \\ \lambda_n \geq 0 \end{array} \right\} \forall g \in \mathbb{R}^n \quad \equiv \quad \exists \lambda_n > 0 \Rightarrow G\lambda_n = 0 \]

Wrench Test

\[ G_n \lambda_n = \begin{bmatrix} f_{1n} \\ f_{2n} \end{bmatrix} = \begin{bmatrix} F_1 - \mathbb{c} \\ F_2 - \mathbb{c} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} = g \]
Wrench Test

\[ G_n \lambda_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} f_{1n} \\ f_{2n} \\ f_{3n} \\ f_{4n} \end{bmatrix} = \begin{bmatrix} f_{1n} - f_{3n} \\ f_{2n} - f_{4n} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} = -g \]

Note that given any \( g \), we can find \( \lambda_n \) such that \( G_n \lambda_n = -g \).

Form closure exists.

Other interpretations:

Velocity constraints
Each contact eliminates a half-space of body twists
Form closure means that no twists are possible

Wrench generation
Non-negative span of columns of \( G_n \) is \( \mathbb{R}^n \)

Geometric interpretation in Wrench Space
Convex hull of columns of \( G_n \) strictly includes origin.

Assume Triangle can only translate in the plane.

\[ \text{nonnegative combinations of columns 2,3} \]

\[ \text{cols 1,2} \]

\[ \text{cols 1,3} \]
Geometric interpretation in workspace (planar only)

If \exists Cones, C_1 \& C_2, formed with pairs of normals, and line segment L connecting the cone apexes \( \exists L \) lies entirely in \( C_1 \cap C_2 \) or \(-C_1 \cap -C_2\), then the grasp has form closure.

Computation Tests for Form Closure.

Linear program \[
\min_x \quad c^T x \\
\text{subject to} \quad A x \geq b
\]

Matlab has good solver.

Form closure requires \( \text{rank}(G) = n_u \) and the existence of \( \lambda_n > 0 \) \( \Rightarrow G_n \lambda_n = 0 \)

Second part

We must satisfy \( G_n \lambda_n = 0 \)
\[ \lambda_n > 0 \]

How do we "encourage" \( \lambda_n \) to be positive?

\[
G_n \lambda_n = 0 \\
\lambda_n - \text{slack} \geq 0 \\implies \lambda_n \geq \text{slack}
\]
slack ≥ 0

Maximize the slack variable

L.P.

\[
\begin{align*}
\max & \quad d \\
\text{s.t.:} & \quad \mathbf{G}_n \mathbf{\lambda}_n = 0 \\
& \quad \mathbf{I} \mathbf{\lambda}_n - \mathbf{d} \geq 0 \\
& \quad \mathbf{d} \geq 0 \\
& \quad \mathbf{1}^T \mathbf{\lambda}_n = n_c
\end{align*}
\]

L is a measure of how far the grasp is from losing form closure

Test for form closure.

1. Rank \( \mathbf{G} = \mathbf{n} \),
   If not, stop. No frictional form closure
2. Compute solution to L.P.
   If \( d > 0 \), then frictional form closure exists!

\[
\mathbf{G}_n = \begin{bmatrix} 1 & -1 \end{bmatrix}
\]

\[
\mathbf{\lambda}_n = \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} = \begin{bmatrix} f_{1n} \\ f_{2n} \end{bmatrix}
\]

\[
\max \quad d
\]

\[
\begin{align*}
\text{s.t.:} & \quad f_{1n} - f_{2n} = 0 \\
& \quad f_{1n} - d \geq 0 \\
& \quad f_{2n} - d \geq 0 \\
& \quad d \geq 0 \\
& \quad f_{1n} + f_{2n} \leq 2
\end{align*}
\]

Feasible Solutions

\[
\begin{align*}
f_{1n} &= f_{2n} & d & \leq f_{1n} \\
0 & & 0 & \\
0.5 & [0, 0.5] & 1 & \text{[0, 1]} \\
1 & [0, 1] & 1.1 & \text{[0, 1.1]}
\end{align*}
\]

Without last constraint, \( f_{1n} \) could increase without bound - infinitely tight squeezing
Planar Example

Figure 28.17

Form closure exists for
\[ 1.052 < \alpha < \frac{\pi}{2} \]

No form closure if \( \alpha = \frac{\pi}{2} \)

See plot in Figure 28.18