Lazy Evaluation:

Infinite data structures, set comprehensions
(CTM Sections 1.8 and 4.5)

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Lazy evaluation

- The functions written so far are evaluated eagerly (as soon as they are called)
- Another way is lazy evaluation where a computation is done only when the result is needed

- Calculates the infinite list:
  0 | 1 | 2 | 3 | ...

```
declare
fun lazy {Ints N}
  N|{Ints N+1}
end
```

Sqrt using an infinite list

```
let sqrt x = head (dropWhile (not . goodEnough) sqrtGuesses)
    where
      goodEnough guess = (abs (x – guess*guess))/x < 0.00001
      improve guess = (guess + x/guess)/2.0
      sqrtGuesses = 1:(map improve sqrtGuesses)
```

Infinite lists (sqrtGuesses) are enabled by lazy evaluation.

Map in Haskell

```
map' :: (a -> b) -> [a] -> [b]
map' _ []    = []
map' f (h:t) = f h:map' f t
```

Functions in Haskell are lazy by default. That is, they can act on infinite data structures by delaying evaluation until needed.

Lazy evaluation (2)

- Write a function that computes as many rows of Pascal's triangle as needed
- We do not know how many beforehand
- A function is *lazy* if it is evaluated only when its result is needed
- The function PascalList is evaluated when needed

```
fun lazy {PascalList Row}
  Row | {PascalList
          {AddList
             {ShiftLeft Row}
             {ShiftRight Row}}}
end
```

Lazy evaluation (3)

- Lazy evaluation will avoid redoing work if you decide first you need the 10th row and later the 11th row
- The function continues where it left off

```
declare
L = {PascalList [1]}
{Browse L}
{Browse L.1}
{Browse L.2.1}
```

```
L<Future>
[1]
[1 1]
```

Lazy execution

- Without lazyness, the execution order of each thread follows textual order, i.e., when a statement comes as the first in a sequence it will execute, whether or not its results are needed later
- This execution scheme is called *eager execution*, or *supply-driven* execution
- Another execution order is that a statement is executed only if its results are needed somewhere in the program
- This scheme is called *lazy evaluation*, or *demand-driven* evaluation (some languages use lazy evaluation by default, e.g., Haskell)

Example

B = {F1 X}

C = {F2 Y}

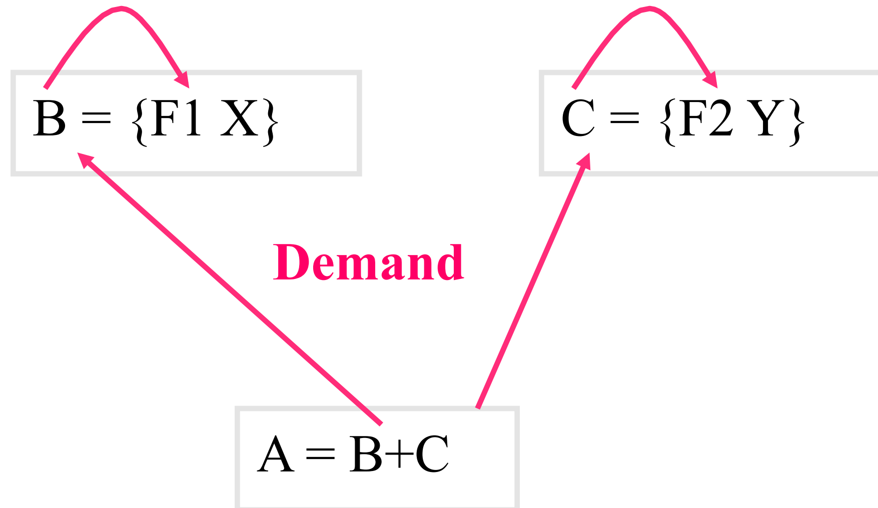
D = {F3 Z}

A = B+C

- Assume F1, F2 and F3 are lazy functions
- B = {F1 X} and C = {F2 Y} are executed only if and when their results are needed in A = B+C
- D = {F3 Z} is not executed since it is not needed

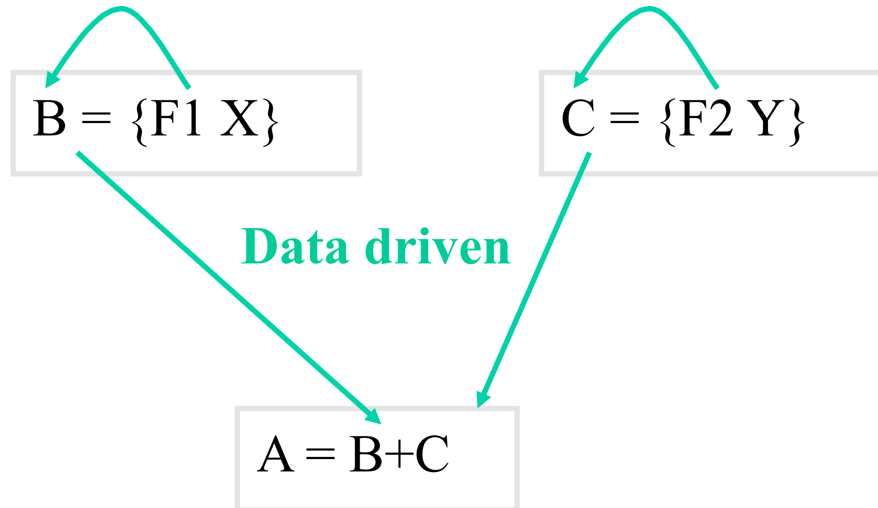
Example

- In lazy execution, an operation suspends until its result is needed
- The suspended operation is triggered when another operation needs the value for its arguments
- In general multiple suspended operations could start concurrently



Example II

- In data-driven execution, an operation suspends until the values of its arguments results are available
- In general the suspended computation could start concurrently



Using Lazy Streams

```
fun {Sum Xs A Limit}
   if Limit>0 then
      case Xs of X|Xr then
         {Sum Xr A+X Limit-1}
      end
   else A end
end
```

```
local Xs S in
   Xs={Ints 0}
   S={Sum Xs 0 1500}
   {Browse S}
end
```

How does it work?

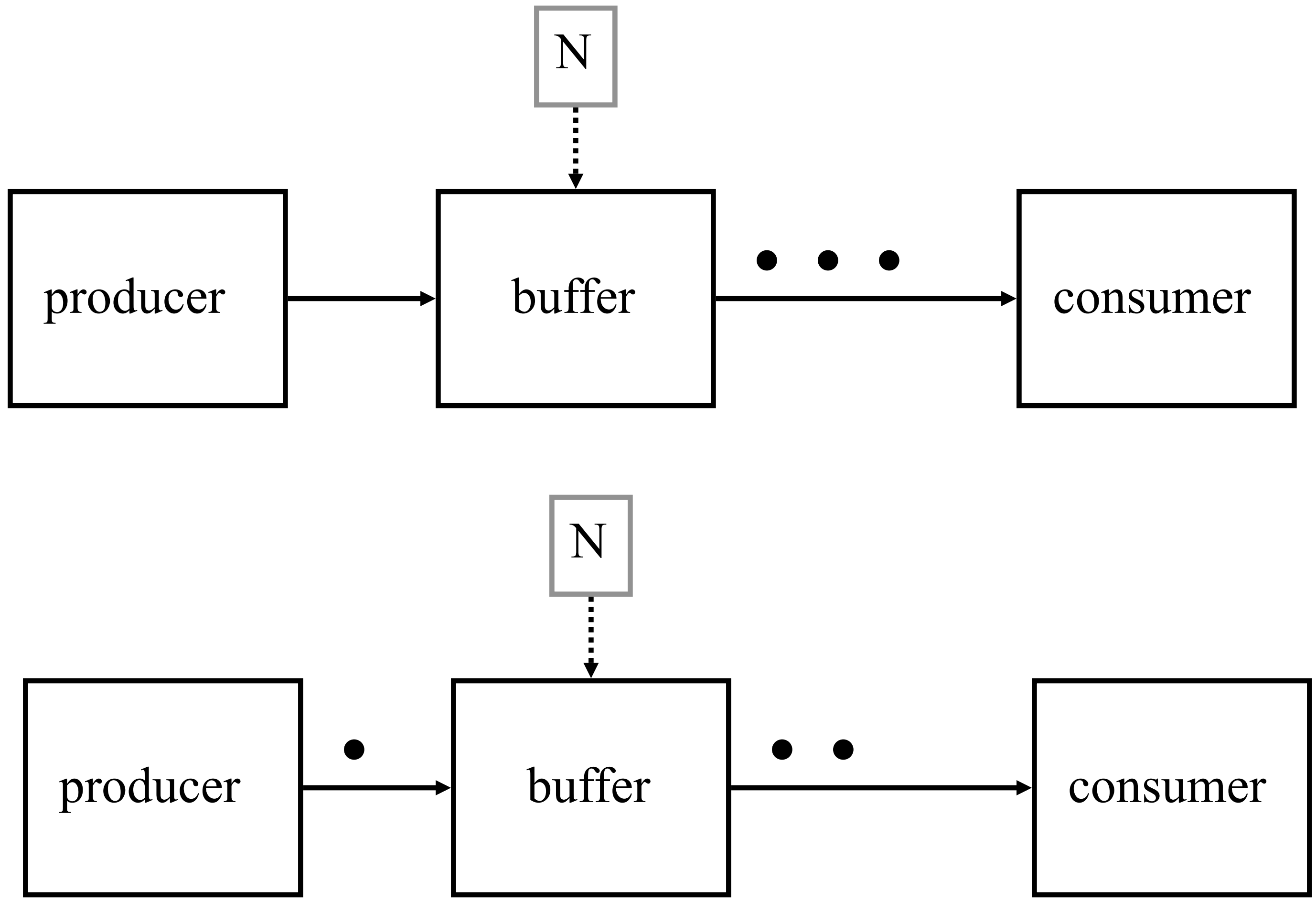
```
fun {Sum Xs A Limit}
   if Limit>0 then
      case Xs of X|Xr then
         {Sum Xr A+X Limit-1}
      end
   else A end
end
```

```
fun lazy {Ints N}
   N | {Ints N+1}
end

local Xs S in
   Xs = {Ints 0}
   S={Sum Xs 0 1500}
   {Browse S}
end
```

Improving throughput

- Use a lazy buffer
- It takes a lazy input stream In and an integer N, and returns a lazy output stream Out
- When it is first called, it first fills itself with N elements by asking the producer
- The buffer now has N elements filled
- Whenever the consumer asks for an element, the buffer in turn asks the producer for another element

The buffer example



The buffer

```
fun {Buffer1 In N}
   End={List.drop In N}

   fun lazy {Loop In End}
      In.1|{Loop In.2 End.2}
   end
in
  {Loop In End}
end
```

Traversing the In stream, forces the producer to emit N elements

The buffer II

```
fun {Buffer2 In N}
   End = thread
            {List.drop In N}
         end
   fun lazy {Loop In End}
      In.1|{Loop In.2 End.2}
   end
in
   {Loop In End}
end
```

Traversing the In stream, forces the producer to emit N elements and at the same time serves the consumer

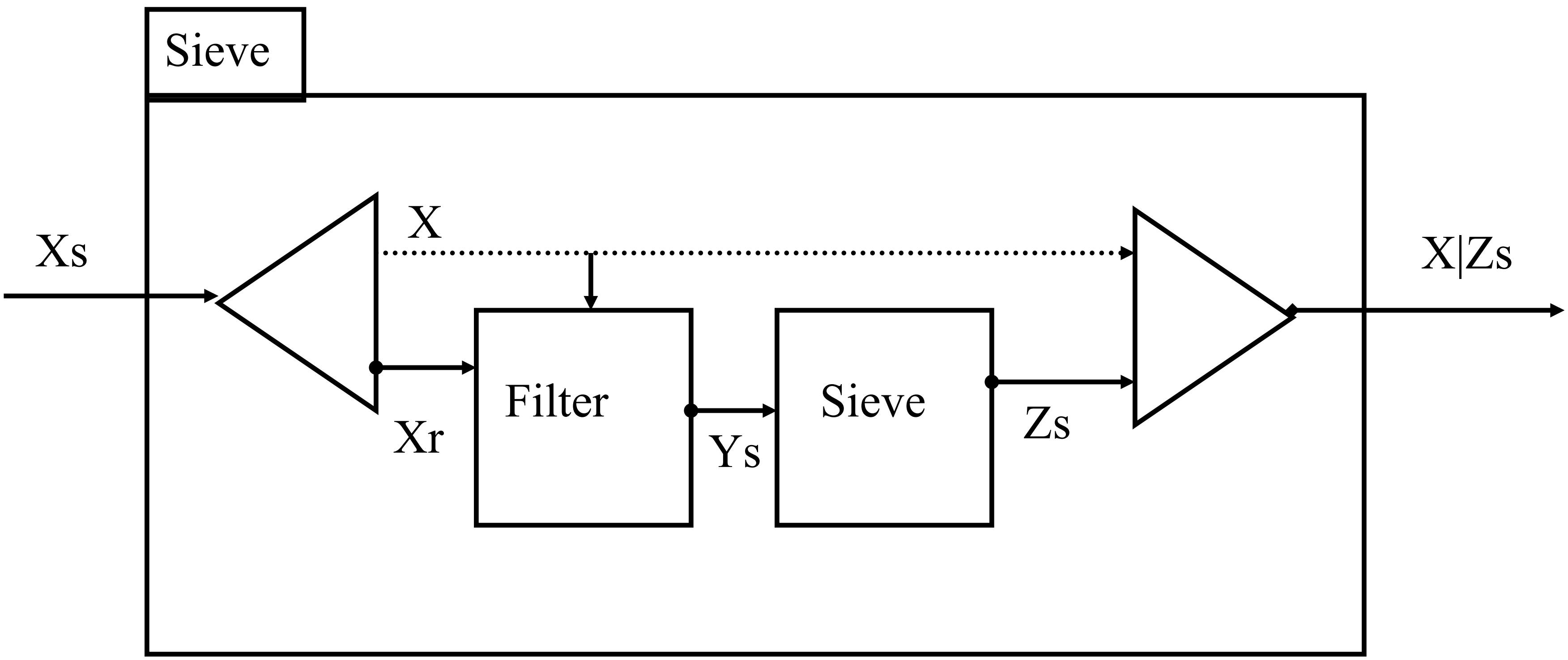
The buffer III

```
fun {Buffer3 In N}
   End = thread
            {List.drop In N}
         end
   fun lazy {Loop In End}
      E2 = thread End.2 end
      In.1|{Loop In.2 E2}
   end
in
   {Loop In End}
end
```

Traverse the In stream, forces the producer to emit N elements and at the same time serves the consumer, and requests the next element ahead

Larger Example: The Sieve of Eratosthenes

- Produces prime numbers
- It takes a stream 2...N, peals off 2 from the rest of the stream
- Delivers the rest to the next sieve



Lazy Sieve

```
fun lazy {Sieve Xs}
   X|Xr = Xs in
   X | {Sieve {LFilter
            Xr
            fun {$ Y} Y mod X \= 0 end
           }}
end

fun {Primes} {Sieve {Ints 2}} end
```

Lazy Filter

For the Sieve program we need a lazy filter

```
fun lazy {LFilter Xs F}
  case Xs
  of nil then nil
  [] X|Xr then
    if {F X} then X|{LFilter Xr F} else {LFilter Xr F} end
  end
end
```

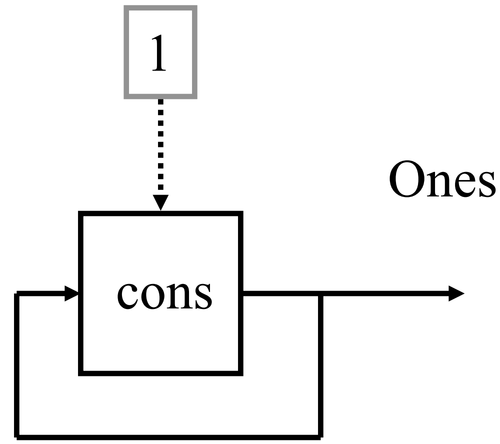
Primes in Haskell

```
ints :: (Num a) => a -> [a]
ints n = n : ints (n+1)

sieve :: (Integral a) => [a] -> [a]
sieve (x:xr) = x:sieve (filter (\y -> (y `mod` x /= 0)) xr)

primes :: (Integral a) => [a]
primes = sieve (ints 2)
```

Functions in Haskell are lazy by default. You can use `take 20 primes` to get the first 20 elements of the list.

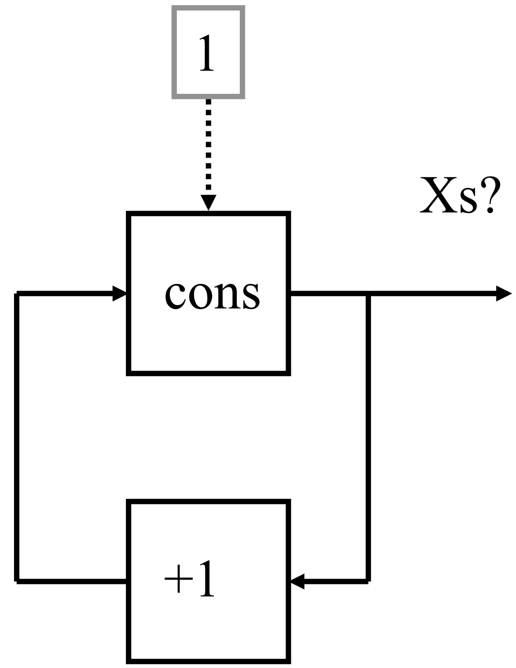
Define streams implicitly

- Ones = 1 | Ones
- Infinite stream of ones



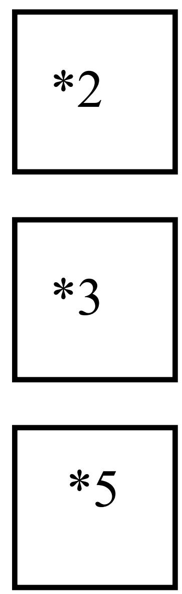
Define streams implicitly

- Xs = 1 | {LMap Xs
  fun {$ X} X+1 end}
- What is Xs ?



The Hamming problem

- Generate the first N elements of stream of integers of the form: $2^a 3^b 5^c$ with a,b,c ≥ 0 (in ascending order)



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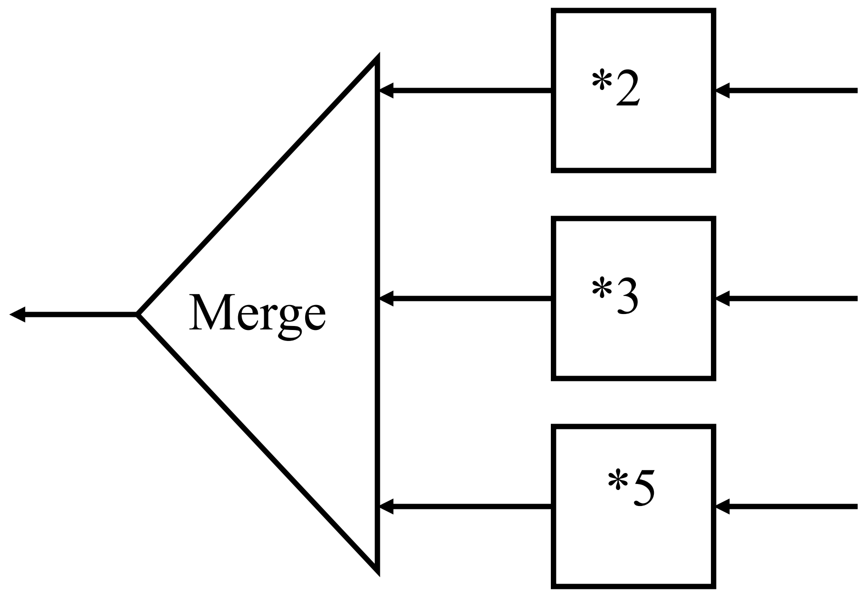

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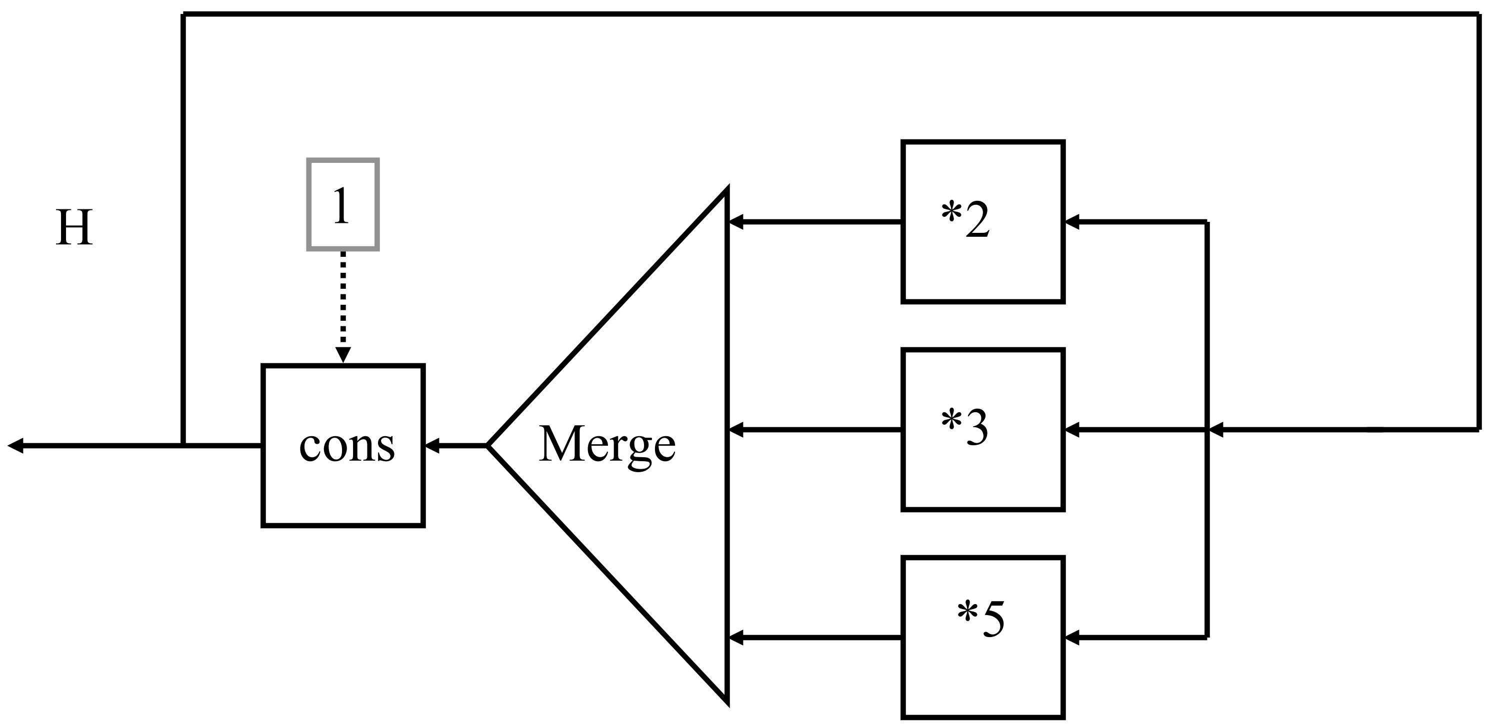


Lazy File Reading

```
fun {ToList FO}
   fun lazy {LRead} L T in
      if {File.readBlock FO L T} then
         T = {LRead}
      else T = nil {File.close FO} end
      L
   end
   {LRead}
end
```

- This avoids reading the whole file in memory

List Comprehensions

- Abstraction provided in lazy functional languages that allows writing higher level set-like expressions
- In our context we produce lazy lists instead of sets
- The mathematical set expression
  - $\{x*y \mid 1 \leq x \leq 10,\ 1 \leq y \leq x\}$
- Equivalent List comprehension expression is
  - [X*Y | X = 1..10 ; Y = 1..X]
- Example:
  - [1*1 2*1 2*2 3*1 3*2 3*3 ... 10*10]

List Comprehensions

- The general form is
- [f(x,y, ...,z) | x ← gen(a1,...,an) ; guard(x,...)
  y ← gen(x, a1,...,an) ; guard(y,x,...)
  ....
  ]
- No linguistic support in Mozart/Oz, but can be easily expressed

Example 1

- $z = [x\#x \mid x \leftarrow from(1,10)]$
- Z = {LMap {LFrom 1 10} fun{$ X} X#X end}

- $z = [x\#y \mid x \leftarrow from(1,10),\ y \leftarrow from(1,x)]$
- Z = {LFlatten

```
Z = {LFlatten
        {LMap {LFrom 1 10}
         fun{$ X} {LMap {LFrom 1 X}
                         fun {$ Y} X#Y end
                        }
         end
        }
    }
```

Example 2

- $z = [x\#y \mid x \leftarrow from(1,10), y \leftarrow from(1,x), x+y \leq 10]$
- Z ={LFilter

```
        {LFlatten
            {LMap {LFrom 1 10}
             fun{$ X} {LMap {LFrom 1 X}
                             fun {$ Y} X#Y end
                           }
             end
            }
         }
        fun {$ X#Y} X+Y=<10 end} }
```

List Comprehensions in Haskell

```
lc1 = [(x,y) | x <- [1..10], y <- [1..x]]

lc2 = filter (\(x,y)->(x+y<=10)) lc1

lc3 = [(x,y) | x <- [1..10], y <- [1..x], x+y<= 10]
```

Haskell provides syntactic support for list comprehensions. List comprehensions are implemented using a built-in list monad.

Quicksort using list comprehensions

```
quicksort :: (Ord a) => [a] -> [a]
quicksort []    = []
quicksort (h:t) = quicksort [x | x <- t, x < h] ++
                  [h] ++
                  quicksort [x | x <- t, x >= h]
```

Higher-order programming

- Higher-order programming = the set of programming techniques that are possible with procedure values (lexically-scoped closures)
- Basic operations
  - Procedural abstraction: creating procedure values with lexical scoping
  - Genericity: procedure values as arguments
  - Instantiation: procedure values as return values
  - Embedding: procedure values in data structures
- Higher-order programming is the foundation of component-based programming and object-oriented programming

Embedding

- Embedding is when procedure values are put in data structures
- Embedding has many uses:
  - Modules: a module is a record that groups together a set of related operations
  - Software components: a software component is a generic function that takes a set of modules as its arguments and returns a new module. It can be seen as specifying a module in terms of the modules it needs.
  - Delayed evaluation (also called explicit lazy evaluation): build just a small part of a data structure, with functions at the extremities that can be called to build more. The consumer can control explicitly how much of the data structure is built.

Explicit lazy evaluation

- Supply-driven evaluation. (e.g.The list is completely calculated independent of whether the elements are needed or not.)
- Demand-driven execution.(e.g. The consumer of the list structure asks for new list elements when they are needed.)
- Technique: a programmed trigger.
- How to do it with higher-order programming? The consumer has a function that it calls when it needs a new list element. The function call returns a pair: the list element and a new function. The new function is the new trigger: calling it returns the next data item and another new function. And so forth.

Explicit lazy functions

```
fun lazy {From N}
   N | {From N+1}
end
```

⇓

```
fun {From N}
   fun {$} N | {From N+1} end
end
```

Implementation of lazy execution

The following defines the syntax of a statement, ⟨s⟩ denotes a statement

$$
\begin{array}{lll}
\langle \mathbf{s} \rangle ::= & \textbf{skip} & \textit{empty statement} \\
| & \ldots & \\
| & \textbf{thread } \langle \mathbf{s}_1 \rangle \textbf{ end} & \textit{thread creation} \\
| & \{\textbf{ByNeed } \underbrace{\textbf{fun}\{\$\}\ \langle \mathbf{e} \rangle\ \textbf{end}}_{\text{zero arity function}}\ \underbrace{\langle x \rangle}_{\text{variable}}\} & \textit{by need statement}
\end{array}
$$

Implementation

some statement

$\{\textbf{ByNeed}\ \underbrace{\textbf{fun}\{\$\}\ \langle \mathbf{e} \rangle\ \textbf{end}}_{\text{f}}\ X,E\ \}$

stack

A function value is created in the
store (say f)
the function *f* is associated with
the variable *x*

store

f

x

execution proceeds immediately to next statement

Implementation

some statement

{ByNeed fun{$} ⟨e⟩ end *X*,*E* }

f

stack

store

f → (**fun**{$} ⟨e⟩ **end** *X*,*E*)

$x : f$

A function value is created in the
the
store (say f)
the function f is **associated**
with
the variable x

execution proceeds
immediately to next statement

Accessing the ByNeed variable

- X = {ByNeed fun{$} 111*111 end} (by thread T0)

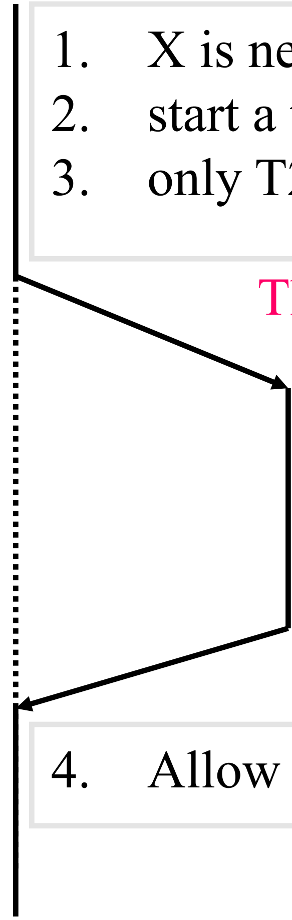
- Access by some thread T1
  - if X > 1000 then {Browse hello#X} end

    or

  - {Wait X}
  - Causes X to be bound to 12321 (i.e. 111*111)

Implementation

Thread T1



1. X is needed
2. start a thread T2 to execute F (the function)
3. only T2 is allowed to bind X

Thread T2

1. Evaluate Y = {F}
2. Bind X the value Y
3. Terminate T2

4. Allow access on X

Lazy functions

```
fun lazy {Ints N}
   N | {Ints N+1}
end
```

```
fun {Ints N}
   fun {F} N | {Ints N+1} end
in {ByNeed F}
end
```

Exercises

26. Write a lazy append list operation `LazyAppend`. Can you also write `LazyFoldL`? Why or why not?
27. CTM Exercise 4.11.10 (pg 341)
28. CTM Exercise 4.11.13 (pg 342)
29. CTM Exercise 4.11.17 (pg 342)
30. Solve exercise 29 (Hamming problem) in Haskell.