Declarative Programming Techniques

Accumulators (CTM 3.4.3)
Difference Lists (CTM 3.4.4)

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November 26, 2019

Accumulators

- *Accumulator programming* is a way to handle state in declarative programs. It is a programming technique that uses arguments to carry state, transform the state, and pass it to the next procedure.
- Assume that the state S consists of a number of components to be transformed individually:

$$S = (X,Y,Z,\ldots)$$

- For each predicate P, each state component is made into a pair, the first component is the *input* state and the second component is the output state after P has terminated
- S is represented as

$$(X_{in}, X_{out}, Y_{in}, Y_{out}, Z_{in}, Z_{out},\ldots)$$

A Trivial Example in Prolog

```
increment(N0,N) :-
    N is N0 + 1.

square(N0,N) :-
    N is N0 * N0.

inc_square(N0,N) :-
    increment(N0,N1),
    square(N1,N).
```

`increment` takes N0 as the input and produces N as the output by adding 1 to N0.

`square` takes N0 as the input and produces N as the output by multiplying N0 to itself.

`inc_square` takes N0 as the input and produces N as the output by using an intermediate variable N1 to carry N0+1 (the output of `increment`) and passing it as input to `square`. The pairs N0-N1 and N1-N are called *accumulators*.

A Trivial Example in Oz

```
proc {Increment N0 N}
    N = N0 + 1
end

proc {Square N0 N}
    N = N0 * N0
end

proc {IncSquare N0 N}
    N1 in
    {Increment N0 N1}
    {Square N1 N}
end
```

`Increment` takes N0 as the input and produces N as the output by adding 1 to N0.

`Square` takes N0 as the input and produces N as the output by multiplying N0 to itself.

`IncSquare` takes N0 as the input and produces N as the output by using an intermediate variable N1 to carry N0+1 (the output of **`Increment`**) and passing it as input to **`Square`**. The pairs N0-N1 and N1-N are called *accumulators*.

Accumulators

- Assume that the state *S* consists of a number of components to be transformed individually:

  $S = (X,Y,Z)$

- Assume P1 to Pn are procedures in Oz

proc $\{P\ \overbrace{X_0\ X}^{\text{accumulator}}\ Y_0\ Y\ Z_0\ Z\}$
$\qquad\vdots$
$\qquad\{P1\ X_0\ X_1\ Y_0\ Y_1\ Z_0\ Z_1\}$
$\qquad\{P2\ X_1\ X_2\ Y_1\ Y_2\ Z_1\ Z_2\}$
$\qquad\vdots$
$\qquad\{Pn\ X_{n-1}\ X\ Y_{n-1}\ Y\ Z_{n-1}\ Z\}$
end

The same concept applies to predicates in Prolog

- The procedural syntax is easier to use if there is more than one accumulator

MergeSort Example

- Consider a variant of MergeSort with accumulator
- proc {MergeSort1 N S0 S Xs}
  - N is an integer,
  - S0 is an input list to be sorted
  - S is the remainder of S0 after the first N elements are sorted
  - Xs is the sorted first N elements of S0
- The pair (S0, S) is an accumulator
- The definition is in a procedural syntax in Oz because it has two outputs S and Xs

Example (2)

```
fun {MergeSort Xs}
   Ys in
   {MergeSort1 {Length Xs} Xs _ Ys}
   Ys
end
```

```
proc {MergeSort1 N S0 S Xs}
   if N==0 then S = S0 Xs = nil
   elseif N ==1 then X in X|S = S0 Xs=[X]
   else %% N > 1
      local S1 Xs1 Xs2 NL NR in
       NL = N div 2
       NR = N - NL
       {MergeSort1 NL S0 S1 Xs1}
       {MergeSort1 NR S1 S Xs2}
       Xs = {Merge Xs1 Xs2}
     end
   end
end
```

MergeSort Example in Prolog

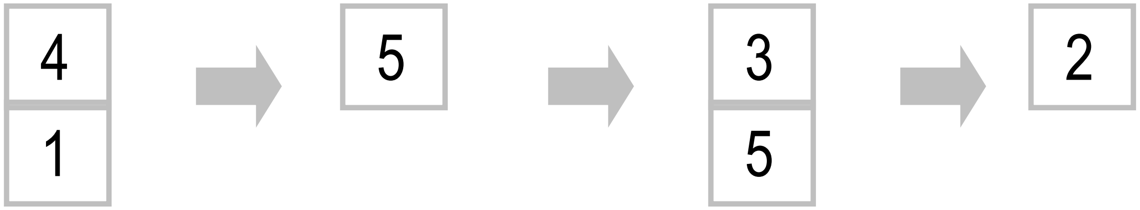
```
mergesort(Xs,Ys) :-
   length(Xs,N),
   mergesort1(N,Xs,_,Ys).
```

```
mergesort1(0,S,S,[]) :- !.
mergesort1(1,[X|S],S,[X]) :- !.
mergesort1(N,S0,S,Xs) :-
        NL is N // 2,
        NR is N - NL,
        mergesort1(NL,S0,S1,Xs1),
        mergesort1(NR,S1,S,Xs2),
        merge(Xs1,Xs2,Xs).
```

Multiple accumulators

- Consider a stack machine for evaluating arithmetic expressions
- Example: (1+4)-3
- The machine executes the following instructions

push(1)
push(4)
plus
push(3)
minus



Multiple accumulators (2)

- Example: (1+4)-3
- The arithmetic expressions are represented as trees:

  `minus(plus(1 4) 3)`

- Write a procedure that takes arithmetic expressions represented as trees and output a list of stack machine instructions and counts the number of instructions

  `proc {ExprCode Expr Cin Cout Nin Nout}`

- Cin: initial list of instructions
- Cout: final list of instructions
- Nin: initial count
- Nout: final count

Multiple accumulators (3)

```
proc {ExprCode Expr C0 C N0 N}
  case Expr
  of plus(Expr1 Expr2) then C1 N1 in
    C1 = plus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] minus(Expr1 Expr2) then C1 N1 in
    C1 = minus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] I andthen {IsInt I} then
    C = push(I)|C0
    N = N0 + 1
  end
end
```

Multiple accumulators (4)

```
proc {ExprCode Expr C0 C N0 N}
  case Expr
  of plus(Expr1 Expr2) then C1 N1 in
    C1 = plus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] minus(Expr1 Expr2) then C1 N1 in
    C1 = minus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] I andthen {IsInt I} then
    C = push(I)|C0
    N = N0 + 1
  end
end
```

```
proc {SeqCode Es C0 C N0 N}
  case Es
  of nil then C = C0 N = N0
  [] E|Er then N1 C1 in
    {ExprCode E C0 C1 N0 N1}
    {SeqCode Er C1 C N1 N}
  end
end
```

Shorter version (4)

```
proc {ExprCode Expr C0 C N0 N}
  case Expr
  of plus(Expr1 Expr2) then
    {SeqCode [Expr2 Expr1] plus|C0 C N0 + 1 N}
  [] minus(Expr1 Expr2) then
    {SeqCode [Expr2 Expr1] minus|C0 C N0 + 1 N}
  [] I andthen {IsInt I} then
    C = push(I)|C0
    N = N0 + 1
  end
end
```

```
proc {SeqCode Es C0 C N0 N}
  case Es
  of nil then C = C0 N = N0
  [] E|Er then N1 C1 in
    {ExprCode E C0 C1 N0 N1}
    {SeqCode Er C1 C N1 N}
  end
end
```

Functional style (4)

```
fun {ExprCode Expr t(C0 N0) }
   case Expr
   of plus(Expr1 Expr2) then
      {SeqCode [Expr2 Expr1] t(plus|C0 N0 + 1)}
   [] minus(Expr1 Expr2) then
      {SeqCode [Expr2 Expr1] t(minus|C0 N0 + 1)}
   [] I andthen {IsInt I} then
      t(push(I)|C0 N0 + 1)
   end
end
```

```
fun {SeqCode Es T}
   case Es
   of nil then T
   [] E|Er then
      T1 = {ExprCode E T} in
      {SeqCode Er T1}
   end
end
```

Difference lists in Oz

- A *difference list* is a pair of lists, each might have an unbound tail, with the invariant that one can get the second list by removing zero or more elements from the first list
- X # X % Represent the empty list
- nil # nil % idem
- [a] # [a] % idem
- (a|b|c|X) # X % Represents [a b c]
- [a b c d] # [d] % idem
- [a b c d|Y] # [d|Y] % idem
- [a b c d|Y] # Y % Represents [a b c d]

Difference lists in Prolog

- A *difference list* is a pair of lists, each might have an unbound tail, with the invariant that one can get the second list by removing zero or more elements from the first list
- X , X % Represent the empty list
- [] , [] % idem
- [a] , [a] % idem
- [a,b,c|X] , X % Represents [a,b,c]
- [a,b,c,d] , [d] % idem
- [a,b,c,d|Y] , [d|Y] % idem
- [a,b,c,d|Y] , Y % Represents [a,b,c,d]

Difference lists in Oz (2)

- When the second list is unbound, an append operation with another difference list takes constant time
-
```
fun {AppendD D1 D2}
        S1 # E1 = D1
        S2 # E2 = D2
in      E1 = S2
        S1 # E2
end
```
- `local X Y in {Browse {AppendD (1|2|3|X)#X (4|5|Y)#Y}} end`
- Displays (1|2|3|4|5|Y)#Y

Difference lists in Prolog (2)

- When the second list is unbound, an append operation with another difference list takes constant time

```
append_dl(S1,E1, S2,E2, S1,E2)  :-  E1 = S2.
```

- `?- append_dl([1,2,3|X],X, [4,5|Y],Y, S,E).`

Displays

```
X = [4, 5|_G193]
Y = _G193
S = [1, 2, 3, 4, 5|_G193]
E = _G193 ;
```

A FIFO queue with difference lists (1)

- A *FIFO queue* is a sequence of elements with an insert and a delete operation.
  - Insert adds an element to the end and delete removes it from the beginning
- Queues can be implemented with lists. If L represents the queue content, then deleting X can remove the head of the list matching X|T but inserting X requires traversing the list {Append L [X]} (insert element at the end).
  - Insert is inefficient: it takes time proportional to the number of queue elements
- With difference lists we can implement a queue with constant-time insert and delete operations
  - The queue content is represented as q(N S E), where N is the number of elements and S#E is a difference list representing the elements

A FIFO queue with difference lists (2)

```
fun {NewQueue} X in q(0 X X) end

fun {Insert Q X}
    case Q of q(N S E) then E1 in E=X|E1 q(N+1 S E1) end
end

fun {Delete Q X}
    case Q of q(N S E) then S1 in X|S1=S q(N-1 S1 E) end
end

fun {EmptyQueue Q} case Q of q(N S E) then N==0 end end
```

- Inserting ‘b’:
  - In: `q(1 a|T T)`
  - Out: `q(2 a|b|U U)`
- Deleting X:
  - In: `q(2 a|b|U U)`
  - Out: `q(1 b|U U)` and `X=a`
- Difference list allows operations at both ends
- N is needed to keep track of the number of queue elements

Flatten

```
fun {Flatten Xs}
case Xs
  of nil then nil
  [] X|Xr andthen {IsLeaf X} then
    X|{Flatten Xr}
  [] X|Xr  andthen {Not {IsLeaf X}} then
    {Append {Flatten X} {Flatten Xr}}
  end
end
```

Flatten takes a list of elements and sub-lists and returns a list with only the elements, e.g.:

{Flatten [1 [2] [[3]]]} = [1 2 3]

Let us replace lists by difference lists and see what happens.

Flatten with difference lists (1)

- Flatten of nil is X#X
- Flatten of a leaf X|Xr is (X|Y1)#Y
  - flatten of Xr is Y1#Y
- Flatten of X|Xr is Y1#Y where
  - flatten of X is Y1#Y2
  - flatten of Xr is Y3#Y
  - equate Y2 and Y3

Here is what it looks like as text

Flatten with difference lists (2)

```
proc {FlattenD Xs Ds}
  case Xs
  of nil then Y in Ds = Y#Y
   [] X|Xr andthen {IsLeaf X} then Y1 Y in
     {FlattenD Xr Y1#Y2}
     Ds = (X|Y1)#Y
   [] X|Xr  andthen {IsList X} then Y0 Y1 Y2 in
     Ds = Y0#Y2
     {FlattenD X Y0#Y1}
     {FlattenD Xr Y1#Y2}
  end
end
fun {Flatten Xs}  Y in {FlattenD Xs Y#nil}  Y end
```

Here is the new program. It is much more efficient than the first version.

Reverse

- Here is our recursive reverse:

```
fun {Reverse Xs}
    case Xs
    of nil then nil
    [] X|Xr then {Append {Reverse Xr} [X]}
    end
end
```

- Rewrite this with difference lists:
  - Reverse of nil is X#X
  - Reverse of X|Xs is Y1#Y, where
    - reverse of Xs is Y1#Y2, and
    - equate Y2 and X|Y

Reverse with difference lists (1)

- The naive version takes time proportional to the square of the input length
- Using difference lists in the naive version makes it linear time
- We use two arguments Y1 and Y instead of Y1#Y
- With a minor change we can make it iterative as well

```
fun {Reverse Xs}
   proc {ReverseD Xs Y1 Y}
      case Xs
      of nil then Y1=Y
      []  X|Xr then Y2 in
         {ReverseD Xr Y1 Y2}
         Y2 = X|Y
      end
   end
   R in
   {ReverseD Xs R nil}
   R
end
```

Reverse with difference lists (2)

```
fun {Reverse Xs}
    proc {ReverseD Xs Y1 Y}
          case Xs
          of nil then Y1=Y
          [] X|Xr then
             {ReverseD Xr Y1 X|Y}
          end
    end
    R in
    {ReverseD Xs R nil}
    R
end
```

Difference lists: Summary

- Difference lists are a way to represent lists in the declarative model such that one append operation can be done in constant time
  - A function that builds a big list by concatenating together lots of little lists can usually be written efficiently with difference lists
  - The function can be written naively, using difference lists and append, and will be efficient when the append is expanded out
- Difference lists are declarative, yet have some of the power of destructive assignment
  - Because of the single-assignment property of dataflow variables
- Difference lists originated from Prolog and are used to implement, e.g., definite clause grammar rules for natural language parsing.

Exercises

91. Rewrite the Oz multiple accumulators example in Prolog.
92. Rewrite the Oz FIFO queue with difference lists in Prolog.
93. Draw the search trees for Prolog queries:
    - `append([1,2],[3],L).`
    - `append(X,Y,[1,2,3]).`
    - `append_dl([1,2|X],X,[3|Y],Y,S,E).`
94. CTM Exercise 3.10.11 (page 232)
95. CTM Exercise 3.10.14 (page 232)
96. CTM Exercise 3.10.15 (page 232)