

Church Numerals

$$\begin{array}{l}
 0 \quad \lambda_f. \lambda_x. x \\
 1 \quad \lambda_f. \lambda_x. (f x) \\
 n \quad \lambda_f. \lambda_x. \underbrace{(f (f \dots (f x) \dots))}_{n \text{ times}}
 \end{array}$$

$$\text{succ} \quad \lambda n. \lambda_f. \lambda_x. (f ((n f) x))$$

$$\begin{array}{l}
 + \quad \lambda m. \lambda n. \lambda_f. \lambda_x. \underbrace{\underbrace{(m f)}_{\underbrace{(f \dots (f x) \dots)}_{N \text{ times}}}}_{\underbrace{(f \dots (f x) \dots)}_{N+M \text{ times}}} \underbrace{((n f) x)}_{\underbrace{(f \dots (f x) \dots)}_{N \text{ times}}}
 \end{array}$$

$$\begin{array}{l}
 * \quad \lambda m. \lambda n. \lambda_f. \lambda_x. \\
 \quad \quad \quad \underbrace{(m \text{ } \cancel{f} \text{ } x)} \\
 \quad \quad \quad \swarrow \quad \searrow \\
 \quad \quad \quad \lambda_x. ((n f) x) \quad \xrightarrow{k} (n f)
 \end{array}$$

$$\lambda m. \lambda n. \lambda_f. \lambda_x. \underbrace{((m (n f)) x)}$$

$$\xrightarrow{k} \lambda m. \lambda n. \lambda_f. (m (n f))$$