

0  $\lambda x. x$   
 1  $\lambda x. \lambda x. x$   
 2  $\lambda x. \lambda x. \lambda x. x$

S  $\lambda n. \lambda x. n$  successor

e.g. (S 0)

$(\lambda n. \underbrace{\lambda x. n}_E \underbrace{\lambda x. x}_M) \xrightarrow{\beta} \lambda x. n \{ \lambda x. x / n \}$   
 $\lambda x. \lambda x. x$   
 1

(S 1)  
 $\xrightarrow{\beta} (\lambda n. \underbrace{\lambda x. n}_E \underbrace{\lambda x. \lambda x. x}_M) \lambda x. n \{ \lambda x. \lambda x. x / n \}$   
 $\lambda x. \lambda x. \lambda x. x$  2

## Add

$$\text{add}(n, m) = \begin{cases} n, & \text{if } m = 0 \\ \text{succ}(\text{add}(n, m')), & \text{if } m = \lambda x. m' \end{cases}$$

$\text{add} = (\lambda a. \lambda n. \lambda m. \text{if } (\text{listen? } m) \text{ } n \text{ } (\text{succ } ((a \ n) \text{ } (\text{pred } m))))$