

$$\begin{aligned}
 & (x \ y) \\
 & (\lambda x. (\overbrace{x \ y}^E) \ \overbrace{\lambda x. x}^M) \\
 \xrightarrow{\beta} & (x \ y) \{ \lambda x. x / x \} \\
 = & (\lambda x. x \ y)
 \end{aligned}$$

$$\begin{aligned}
 e ::= & \ v \\
 & \mid \ \lambda v. e \\
 & \mid \ (e \ e)
 \end{aligned}$$

$$\begin{aligned}
 & (\lambda x. E \ M) \\
 \xrightarrow{\beta} & E \{ M / x \}
 \end{aligned}$$

$$\begin{aligned}
 & \xrightarrow{\beta} y \\
 & (\lambda x. (\overbrace{\lambda x. x \ y}^E) \ \overbrace{\lambda x. x}^M) \\
 \xrightarrow{\beta} & (\lambda x. x \ y) \{ \lambda x. x / x \} \\
 = & (\lambda x. x \ y) \\
 & (\lambda x. (\overbrace{\lambda x. x \ x}^E) \ \overbrace{\lambda x. x}^M) \\
 & \quad \underbrace{\lambda x. x}_{\lambda x. x} \quad \underbrace{\lambda x. x}_{\lambda x. x}
 \end{aligned}$$

$$(x \ y \ z)$$

$$(x \ ; \ y)$$

$$(x \leftarrow y)$$

$$x \ y$$

$$(\overbrace{x}^{\lambda x. x})$$

$$x. \lambda x. x$$

$$\begin{aligned}
 \xrightarrow{\beta} & (\lambda x. x \ x) \{ \lambda x. x / x \} \\
 = & (\lambda x. x \ \overbrace{\lambda x. x}^M) \\
 \xrightarrow{\beta} & x \{ \lambda x. x / x \} \\
 = & \lambda x. x
 \end{aligned}$$

$$(\lambda x. (\overbrace{x \ x}^E) \ \overbrace{\lambda y. y}^M)$$

$$\xrightarrow{\beta} (x \ x) \{ \lambda y. y / x \}$$

$$= (\lambda y. y \ \lambda y. y)$$

$$\xrightarrow{\beta} \lambda y. y$$