

Church Numerals

$$0 \quad \lambda f. \lambda x. x$$

$$1 \quad \lambda f. \lambda x. (f x)$$

$$n \quad \lambda f. \lambda x. \underbrace{(f (f \dots (f x) \dots))}_{n \text{ times}}$$

$$\text{succ} \quad \lambda n. \lambda f. \lambda x. (f ((n f) x))$$

$$+ \quad \lambda m. \lambda n. \lambda f. \lambda x. \underbrace{(\underbrace{(m f)}_{\text{M times}}) \underbrace{((n f) x)}_{\text{N times}})}_{\text{N+M times}}$$

$$\begin{aligned} * : \quad & \lambda m. \lambda n. \lambda f. \lambda x. \\ & \quad \quad \quad \underbrace{((m f) x)}_{\substack{\lambda x. ((n f) x) \\ \xrightarrow{h} (n f)}} \end{aligned}$$

$$\lambda m. \lambda n. \lambda f. \lambda x. \underbrace{((m (n f)) x)}$$

$$\xrightarrow{h} \lambda m. \lambda n. \lambda f. (m (n f))$$

$$\wedge : \quad \lambda m. \lambda n. (n m)$$

$$\text{exp2} : \quad \lambda n. (n \lambda f. \lambda x. (f (f x)))$$

$$\begin{aligned} \text{square} : \quad & \lambda n. (\lambda f. \lambda x. (f (f x)) n) \\ & \xrightarrow{h} \lambda f. \lambda x. (f (f x)) \end{aligned}$$