

Church Numerals (Factorial)

$$\text{fact } n = \begin{cases} 1, & \text{if } n=0 \\ n * \text{fact } (n-1), & \text{o/w.} \end{cases}$$

$$\text{fact } (\lambda f. \lambda n. (((\text{isZero? } n) \lambda x. x) ((*) n) (f (\text{pred } n)))))$$

where:

$$\text{isZero? } \lambda n. \lambda t. \lambda f. ((n \lambda x. f) t)$$

$$* \lambda m. \lambda n. \lambda f. (m (n f))$$

$$\text{pred: } \lambda n. \lambda f. \lambda x. (2^{\text{nd}} ((n h) ((pr \lambda x. x) x)))$$

$$\text{where } h: \lambda p. ((pr f) ((1^{\text{st}} p) (2^{\text{nd}} p)))$$

$$pr: \lambda x. \lambda y. \lambda b. ((b x) y)$$

$$1^{\text{st}}: \lambda p. (p \lambda x. \lambda y. x)$$

$$2^{\text{nd}}: \lambda p. (p \lambda x. \lambda y. y)$$