ε-transitions of FA

Nondeterministic automata and algorithms, though counter-intuitive, are useful models for designing and analyzing real machines. In this topic we will study more about the **e**-transitions in FA, which make the machines nondeterministic. Consider a transition between two states shown in Figure (a) below, which means that reding input symbol *b*, the machine changes its state from q_1 to q_2. We assume that if there is **no input**, the FA remains in current state q_1. Let ε denote null (i.e., no) input. If we explicitly show the null transitions for the states in Figure (a), we will get Figure (b), which is equivalent to Figure (a).

Now, suppose that there is an **e**-transition to a different state as shown in Figure (c), the automaton is nondeterministic, because it has two transitions for the same input **e, i.e.,** $\delta(q_1, \varepsilon) = \{q_1, q_2\}$. Therefore, if an FA has an **e**-transition between two different states, then the automaton is nondeterministic.

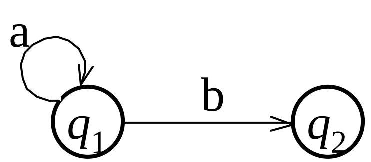


Figure (a)

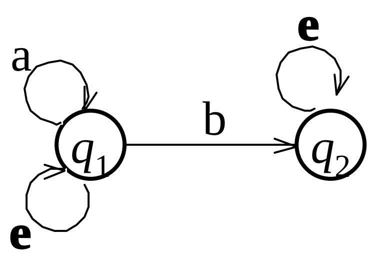


Figure (b)

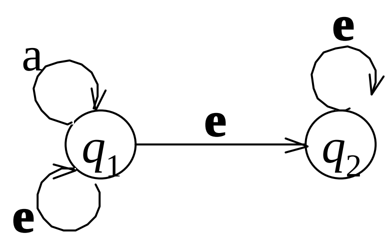


Figure (c)

Eliminating ε-transitions from an FA

If an FA has ε-transitions, we can eliminate all of them from the machine by adding some non-ε-transitions such that the resulting machine accepts the same language with the same number of states. Before we show how, we need to introduce some notational convention.

Recall that for a state q and a symbol a, $\delta(q, a)$ is the set of next states reachable from state q with input a. (If the automaton is deterministic, there is only one reachable state.) We generalize this definition. For a set of states A and a string x, let $\delta(A, x)$ denote the set of states the automaton enters from all the states in A after reading string x. Thus, if x = yz, then clearly,

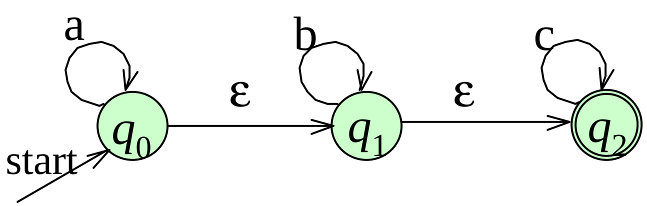
$$\delta(q, x) = \delta(q, yz) = \delta(\ \delta(q, y), z)$$

i.e., the set of states reachable from state q with string x is the states reachable from all the states, that are reachable by transition $\delta(q, y)$, with string z.

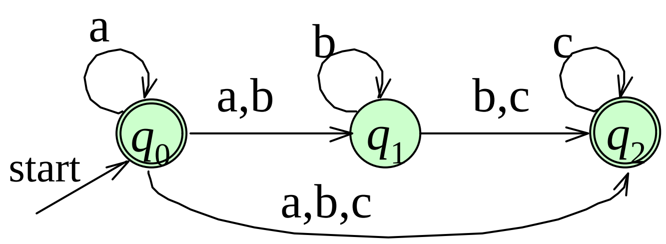
Another notational convention that we want to use for eliminating ε-transitions; since $\varepsilon\varepsilon = \varepsilon$, we have $\varepsilon^* = \varepsilon$. For a symbol **a**, since $\mathbf{a} = \varepsilon\mathbf{a}\varepsilon$, we can write $\mathbf{a} = \varepsilon^*\mathbf{a}\,\varepsilon^*$.

Eliminating ε-transitions (cont’ ed)

Let δ be the transition function of an FA M with ε-transitions as shown in Figure (a) below. We construct the transition function δ' of M' which has no ε-transitions as shown in Figure (b). (Notice that M’ is still an NFA.)



(a) FA with ε-transitions



(b) FA with no ε-transitions

The idea is, for each state q_i and each input symbol t (in {a, b, c} in the above FA), to compute the set of states reachable by $\delta'(q_i, t)$ in M' by collect all reachable states in M from q_i for input $\varepsilon^* t \varepsilon^*$, i.e., to compute $\delta(q_i, \varepsilon^* t \varepsilon^*)$. From the state transition graph of M, we can easily collect such state set. For example,

$$\delta'(q_0, a) = \delta(q_0, \varepsilon^* a \varepsilon^*) = \delta(\delta(q_0, \varepsilon^*), a\varepsilon^*) = \delta(\{q_0, q_1, q_2\}, a\varepsilon^*) =$$
$$\delta(\delta(q_0, q_1, q_2\}, a), \varepsilon^*) = \delta(q_0, \varepsilon^*) = \{q_0, q_1, q_2\}.$$

Eliminating ε-transitions (cont' ed)

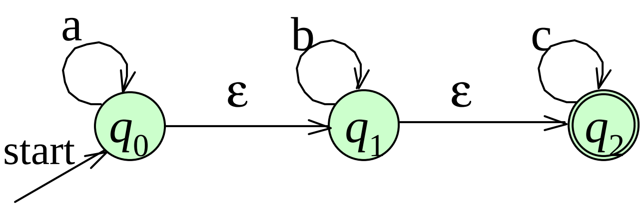
Using the same technique for other transitions, we get the following results:

$\delta'(q_0, a) = \{ q_0, q_1, q_2 \}$

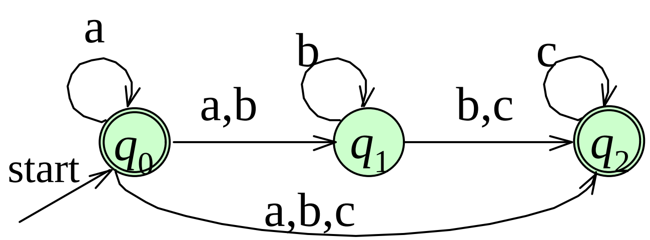
$\delta'(q_0, b) = \{ q_1, q_2 \}$, $\delta'(q_0, c) = \{ q_2 \}$, $\delta'(q_1, a) = \varnothing$, $\delta'(q_1, b) = \{ q_1, q_2 \}$,

$\delta'(q_1, c) = \{ q_2 \}$, $\delta'(q_2, a) = \varnothing$, $\delta'(q_2, b) = \varnothing$, $\delta'(q_2, c) = \{ q_2 \}$.

The following graph in Figure (b) below represents these transitions.



(a) FA with ε-transitions



(b) FA with no ε-transitions