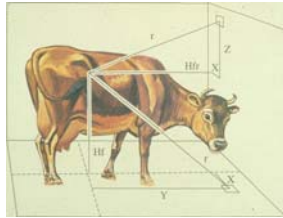


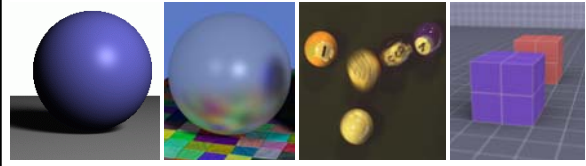
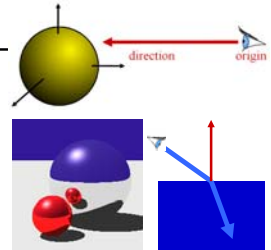
## Local vs. Global Illumination & Radiosity



An early application of radiative heat transfer in stables.

## Last Time?

- Ray Casting & Ray-Object Intersection
- Recursive Ray Tracing
- Distributed Ray Tracing

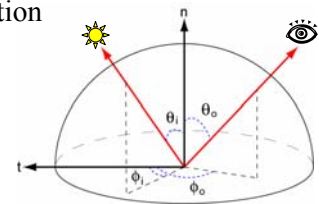


## Today

- **Local Illumination**
  - BRDF
  - Ideal Diffuse Reflectance
  - Ideal Specular Reflectance
  - The Phong Model
- Why is Global Illumination Important?
- Radiosity Equation/Matrix
- Calculating the Form Factors

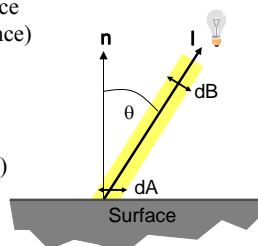
## BRDF

- Ratio of light coming from one direction that gets reflected in another direction
- Bidirectional Reflectance Distribution Function
  - 4D
  - $R(\theta_i, \phi_i; \theta_o, \phi_o)$



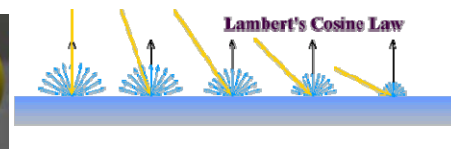
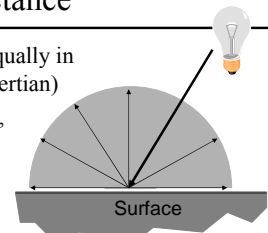
## Incoming Radiance

- The amount of light received by a surface depends on incoming angle
  - Bigger at normal incidence (Winter/Summer difference)
- By how much?
  - $dB = dA \cos \theta$
  - Same as:  $\mathbf{l} \cdot \mathbf{n}$  (dot product with normal)



## Ideal Diffuse Reflectance

- Assume surface reflects equally in all directions (a.k.a. Lambertian)
- An ideal diffuse surface is, at the microscopic level, a very rough surface
- Examples: chalk, clay, some paints



### Ideal Specular Reflectance

- Assume surface reflects only in mirror direction
  - View dependent
- Microscopic surface elements are oriented in the same direction as the surface
- Examples: mirrors, highly polished metals

### Non-Ideal Reflectors

- Real materials tend to be *neither* ideal diffuse *nor* ideal reflective
- Highlight is blurry, looks glossy

### Non-Ideal Reflectors

- Most light reflects in the ideal reflected direction
- Microscopic surface variations will reflect light just slightly offset
- How much light is reflected?

### The Phong Model

- How much light is reflected “specularly”?
  - Depends on the angle between the ideal reflection direction and the viewer direction  $\alpha$ .

$$L_o = k_s (\cos \alpha)^q \frac{L_i}{r^2}$$

$k_s$ : specular reflection coefficient  
 $q$ : specular reflection exponent

### The Phong Model

- Sum of three components:
  - diffuse reflection + specular reflection + “ambient”.

variations in Phong specular exponent

### Ambient Illumination

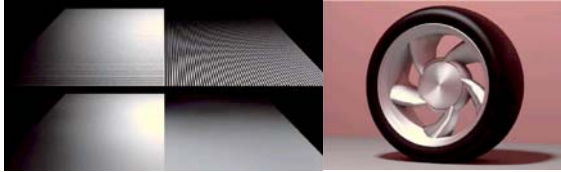
- In a typical room, everything receives at least a little bit of light
- Ambient illumination represents the reflection of all indirect illumination

$$L(\omega_r) = k_a$$

- This is a total hack!

## Anisotropic BRDFs

- Surfaces with strongly oriented microgeometry
- Examples:
  - brushed metals, hair, fur, cloth, velvet



Source: Westin et.al 92

## Questions?



Lightscape <http://www.lightscape.com>

## Today

- Local Illumination
- **Why is Global Illumination Important?**
  - **The Cornell Box**
  - **Radiosity vs. Ray Tracing**
- Radiosity Equation/Matrix
- Calculating the Form Factors

## Why Global Illumination?

- Simulate all light inter-reflections (indirect lighting)
  - in a room, a lot of the light is indirect: it is reflected by walls.
- How have we dealt with this so far?
  - Ambient term to fake some uniform indirect light



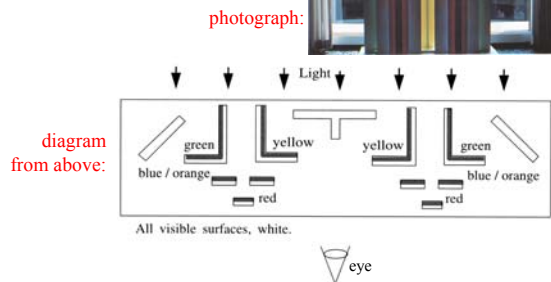
(no ambient term)

it is smooth, but not constant!

Henrik Wann Jensen

## Why Radiosity?

- Sculpture by John Ferren
- *Diffuse* panels



## Radiosity vs. Ray Tracing

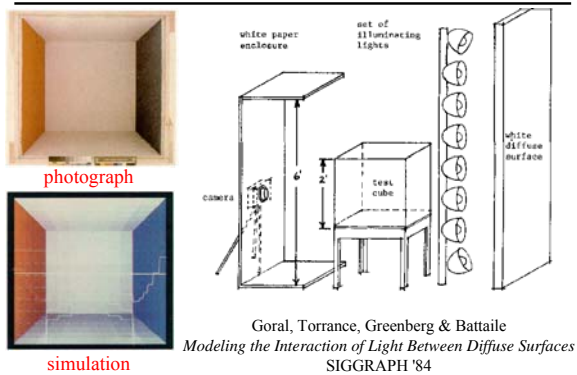


Original sculpture by John Ferren lit by daylight from behind.

Ray traced image. A standard ray tracer cannot simulate the interreflection of light between diffuse surfaces.

Image rendered with radiosity. note color bleeding effects.

## Reading for Today:



## The Cornell Box

- Careful calibration and measurement allows for comparison between physical scene & simulation



Light Measurement Laboratory  
Cornell University, Program for Computer Graphics

## Two approaches for global illumination

- Radiosity
  - View-independent
  - Diffuse materials only
- Monte-Carlo Ray-tracing
  - Send tons of indirect rays

## Visualizing Inter-reflections...

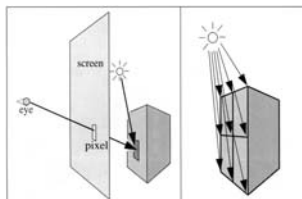


images by Micheal Callahan

[http://www.cs.utah.edu/~shirley/classes/cs684\\_98/students/callahan/bounce/](http://www.cs.utah.edu/~shirley/classes/cs684_98/students/callahan/bounce/)

## Radiosity vs. Ray Tracing

- Ray tracing is an *image-space* algorithm
  - If the camera is moved, we have to start over
- Radiosity is computed in *object-space*
  - View-independent (just don't move the light)
  - Can pre-compute complex lighting to allow interactive walkthroughs

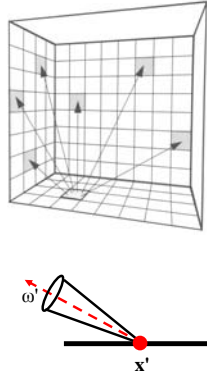


## Today

- Local Illumination
- Why is Global Illumination Important?
- Radiosity Equation/Matrix**
- Calculating the Form Factors

## Radiosity Overview

- Surfaces are assumed to be perfectly Lambertian (diffuse)
  - reflect incident light in all directions with equal intensity
- The scene is divided into a set of small areas, or patches.
- The radiosity,  $B_i$ , of patch  $i$  is the total rate of energy leaving a surface. The radiosity over a patch is constant.
- Units for radiosity: Watts / steradian \* meter<sup>2</sup>

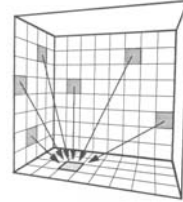


## Radiosity Equation

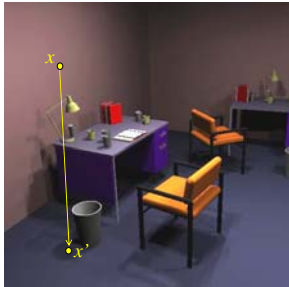
$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

Radiosity assumption:  
perfectly diffuse surfaces (not directional)

$$B_{x'} = E_{x'} + \rho_{x'} \int B_x G(x, x') V(x, x')$$



## Continuous Radiosity Equation



$$B_{x'} = E_{x'} + \rho_{x'} \int G(x, x') V(x, x') B_x$$

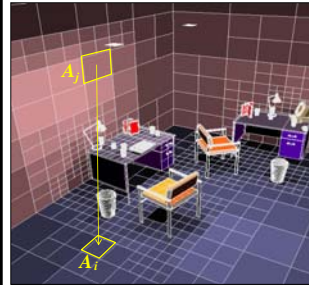
reflectivity  
↓  
form factor

G: geometry term  
V: visibility term

No analytical solution, even for simple configurations

## Discrete Radiosity Equation

Discretize the scene into  $n$  patches, over which the radiosity is constant



$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

reflectivity  
↓  
form factor

- discrete representation
- iterative solution
- costly geometric/visibility calculations

## The Radiosity Matrix

$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

$n$  simultaneous equations with  $n$  unknown  $B_i$  values can be written in matrix form:

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & & \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & \dots & \dots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

A solution yields a single radiosity value  $B_i$  for each patch in the environment, a view-independent solution.

## Solving the Radiosity Matrix

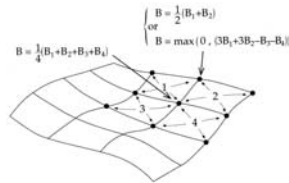
The radiosity of a single patch  $i$  is updated for each iteration by gathering radiosities from all other patches:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} \rho_1 F_{1i} & \rho_1 F_{2i} & \dots & \rho_1 F_{ni} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix}$$

This method is fundamentally a Gauss-Seidel relaxation

## Computing Vertex Radiosities

- $B_i$  radiosity values are constant over the extent of a patch.
- How are they mapped to the vertex radiosities (intensities) needed by the renderer?
  - Average the radiosities of patches that contribute to the vertex
  - Vertices on the edge of a surface are assigned values extrapolation



## Questions?



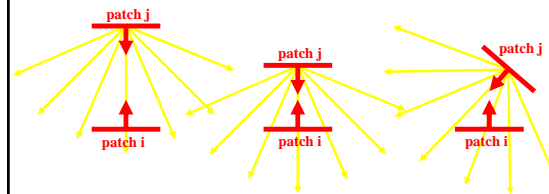
Factory simulation. Program of Computer Graphics, Cornell University. 30,000 patches.

## Today

- Local Illumination
- Why is Global Illumination Important?
- The Rendering Equation
- Radiosity Equation/Matrix
- **Calculating the Form Factors**

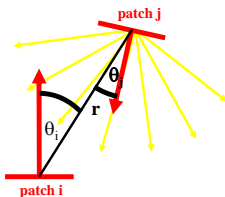
## Calculating the Form Factor $F_{ij}$

- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i
- Takes account of both:
  - geometry (size, orientation & position)
  - visibility (are there any occluders?)



## Calculating the Form Factor $F_{ij}$

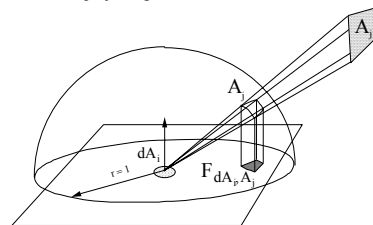
- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i



$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} V_{ij} dA_j dA_i$$

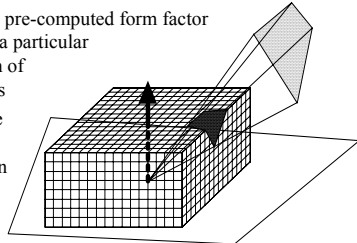
## Form Factor Determination

The Nusselt analog: the form factor of a patch is equivalent to the fraction of the unit circle that is formed by taking the projection of the patch onto the hemisphere surface and projecting it down onto the circle.



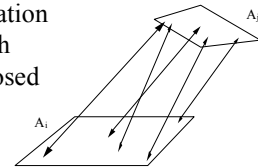
## Hemicube Algorithm

- A hemicube is constructed around the center of each patch
- Faces of the hemicube are divided into "pixels"
- Each patch is projected (rasterized) onto the faces of the hemicube
- Each pixel stores its pre-computed form factor  
The form factor for a particular patch is just the sum of the pixels it overlaps
- Patch occlusions are handled similar to z-buffer rasterization



## Form Factor from Ray Casting

- Cast  $n$  rays between the two patches
  - $n$  is typically between 4 and 32
  - Compute visibility
  - Integrate the point-to-point form factor
- Permits the computation of the patch-to-patch form factor, as opposed to point-to-patch



## Questions?



Lightscape <http://www.lightscape.com>

## Reading for Tuesday 3/4:

- "A Two-Pass Solution to the Rendering Equation: A Synthesis of Ray Tracing and Radiosity Methods"  
Wallace, Cohen, & Greenberg, SIGGRAPH 1987



- *Optional Reading*: "The Rendering Equation"  
Kajiya, SIGGRAPH 1986