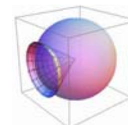
Modeling the Interaction Between Diffuse Surfaces

Some Definitions

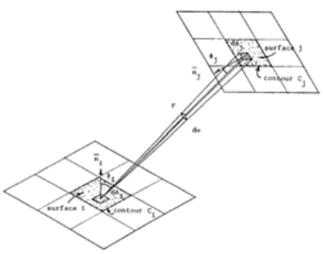
- Solid Angle: The surface area of the unit sphere covered by the projection of the angle from the center of the sphere.

Weisstein, Eric W. "Solid Angle." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/SolidAngle.html



- Isolux: A curve or surface connecting points at which light intensity is the same.
- Form Factor: The fraction of radiant light leaving one surface which strikes second surface

The Basic Idea



dA(i) = elemental area on surface I

A(i) = area of surface I

C(i) = contour of surface I

r = distance between dA(i) and dA(j)

dω = solid angle subtended by dA(j) as seen from dA(i)

Φ(i) = angle between surface normal of i, n(i), and the line r

Φ(j) = angle between surface normal of j, E(j), and the line r

dA(j) = elemental area on surface j

A(j) = area of surface j

C(j) = contour of surface j

Modeling the Interaction of Light Between Diffuse Surfaces: Figure 5

Some Math

$$B_j = E_j + \rho_j H_j$$

Where: B_j = radiosity of the surface j
E_j = rate of direct energy emission from j
ρ_j = reflectivity of surface j (fraction of incident light reflected back)
H_j = incident radiant energy (light) arriving at j

$B_j = E_j + \rho_j H_j$

$$H_j = \sum_{i=1}^{N} B_i F_{ij}$$

Where: B_j = radiosity of the surface i
F_{ij} = form factor

Some Math (cont)

Final radiosity equation

$$b_j = e_j + \rho_j \sum_{i=1}^{N} b_j F_{ij}$$

Form Factors

The form factor between the finite surfaces, A(i) and A(j), is defined as the area average of the equation:

$$F_{A_i - A_j} = F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\phi_i \cos\phi_j dA_i dA_j}{\pi r^2}$$

Or more efficiently as:

$$F_{ij} = \frac{1}{2\pi A_i} \oint_{C_j} \oint_{C_i} [\ln(r) dx_i dx_j + \ln(r) dy_i dy_j + \ln(r) dz_i dz_j]$$

Why? Stokes' theorem?

And Now Some Matrixes

The final system to solve:

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & -\rho_1 F_{1,2} & \dots & -\rho_1 F_{1,N} \\ -\rho_2 F_{2,1} & 1-\rho_1 F_{2,2} & \dots & -\rho_2 F_{2,N} \\ \dots & \dots & \dots & \dots \\ -\rho_N F_{N,1} & -\rho_N F_{N,2} & \dots & 1-\rho_N F_{N,N} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ \dots \\ e_N \end{bmatrix}$$