

A Two-Pass Solution to the Rendering Equation:
A Synthesis of Ray Tracing and Radiosity
Methods

Wallace, Cohen, and Greenberg
1987

Rendering Equation

$$I_{\text{out}}(\theta_{\text{out}}) = E(\theta_{\text{out}}) + \int_{\Omega} \rho''(\theta_{\text{out}}, \theta_{\text{in}}) I_{\text{in}}(\theta_{\text{in}}) \cos(\theta) d\omega$$

I_{out} = the outgoing intensity for the surface

I_{in} = an intensity arriving at the surface from the rest of the environment

E = outgoing intensity due to emission by the surface

θ_{out} = the outgoing direction

θ_{in} = the incoming direction

Ω = the sphere of incoming directions

θ = the angle between the incoming direction θ_{in} and the surface normal

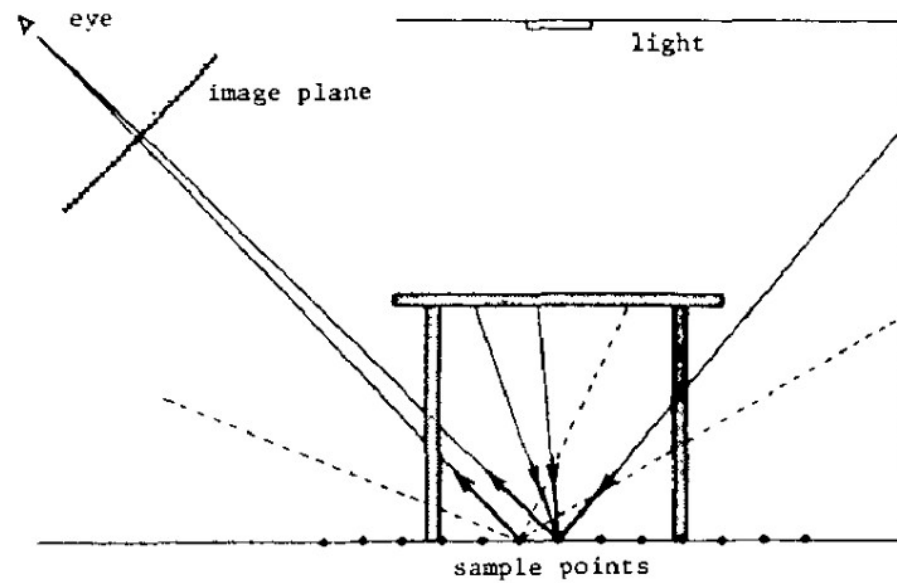
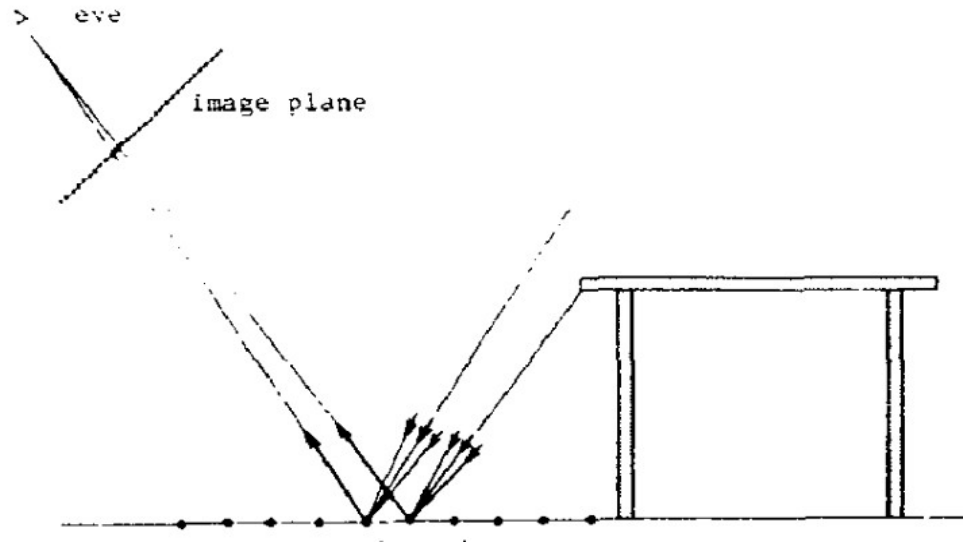
$d\omega$ = the differential solid angle through which the incoming intensity arrives

ρ'' = the bidirectional reflectance/transmittance of the surface

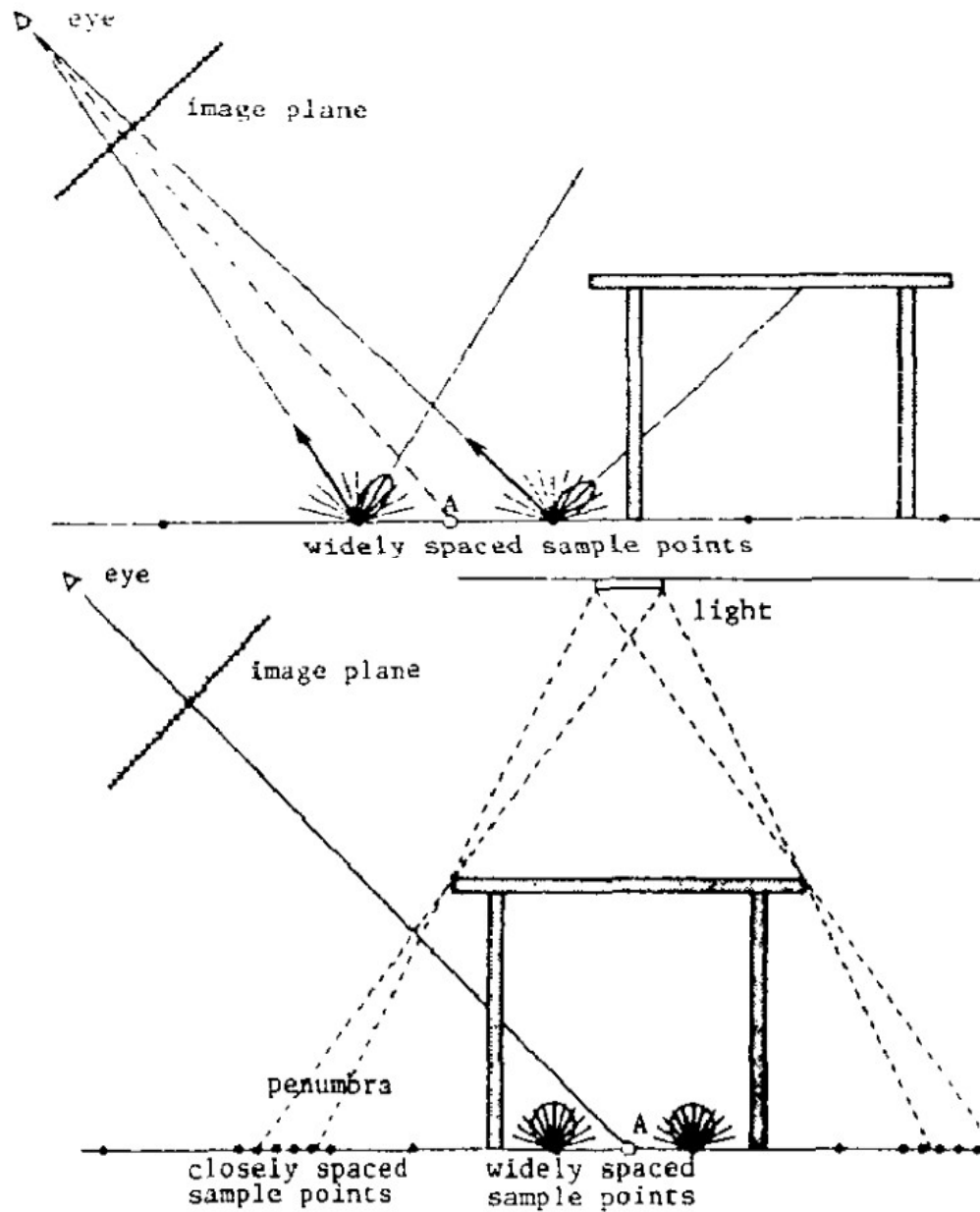
Previous Methods

- View Dependent Ray Tracing
 - Captures specular reflection
- View Independent Radiosity
 - Captures diffusion
- Both can be extended to account for the other

Enhanced Ray Tracing



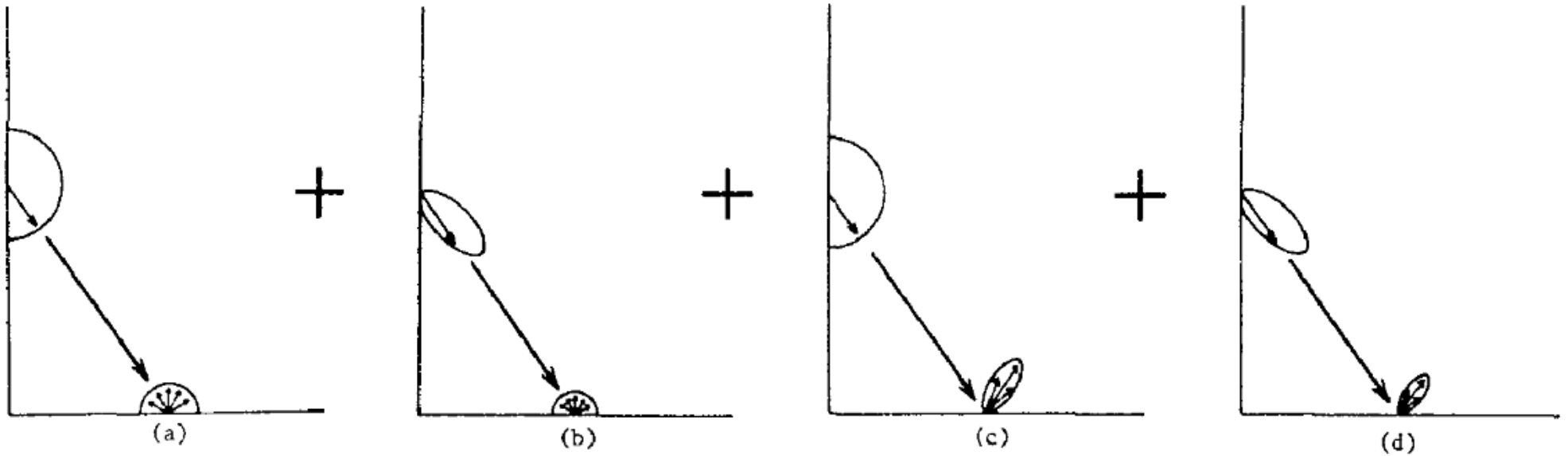
Enhanced Radiosity



Downsides of Each

- Ray Tracing:
 - Lighting in areas which are mostly diffuse surfaces does not change often
- Radiosity:
 - Patches become very small in areas with a high lighting gradient (specular reflections)

Types of Transmission



Algorithm Foundation

- Break BRDF down into specular and diffuse parts

$$\rho''(\theta_{out}, \theta_{in}) = k_s \rho_s(\theta_{out}, \theta_{in}) + k_d \rho_d$$

where

k_s = fraction of reflectance that is specular

k_d = fraction of reflectance that is diffuse

$$k_s + k_d = 1$$

$$I_{out}(\theta_{out}) = E(\theta_{out}) + I_{d,out} + I_{s,out}(\theta_{out})$$

$$I_{d,out} = k_d \rho_d \int I_{in}(\theta_{in}) \cos(\theta) d\omega$$

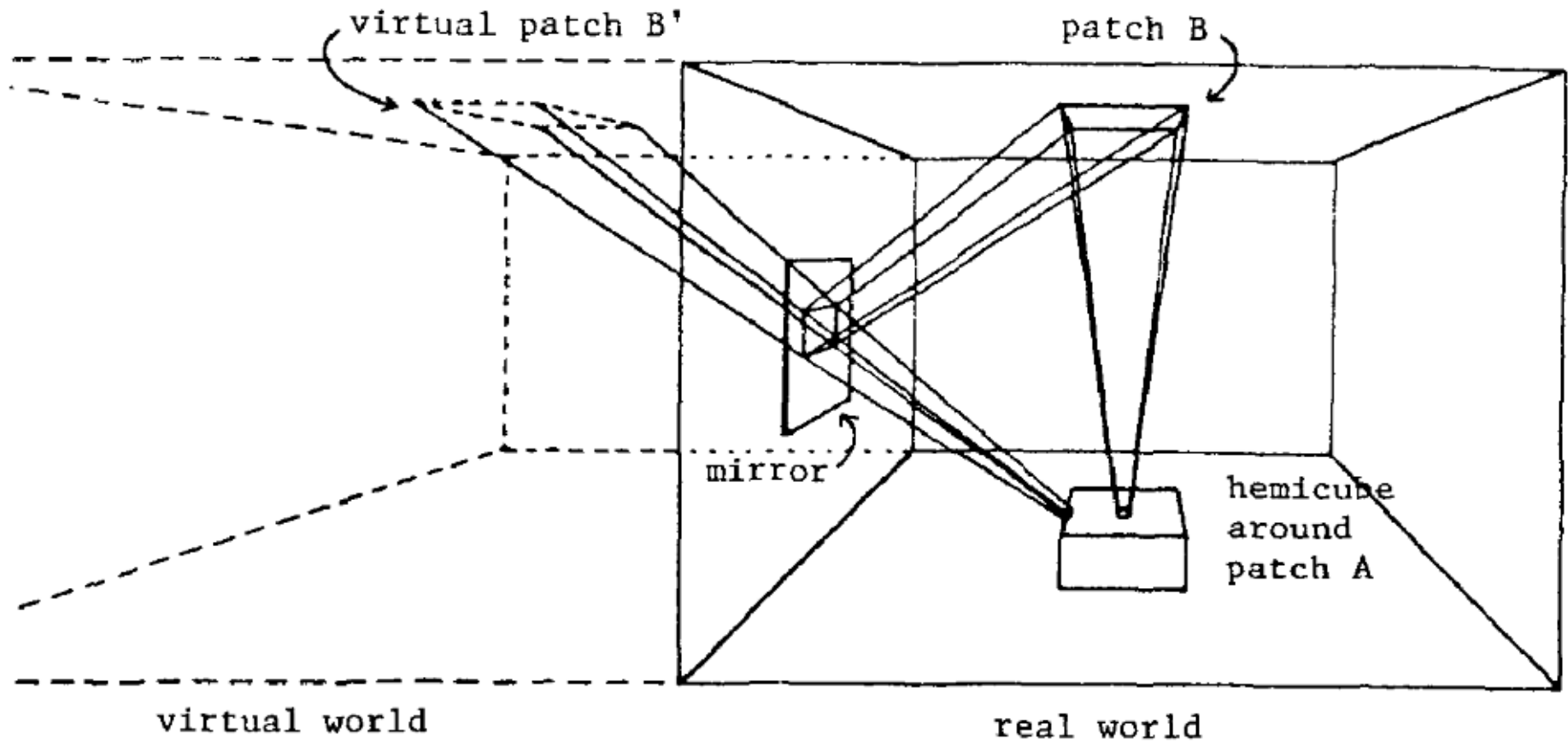
$$I_{s,out}(\theta_{out}) =$$

$$k_s \int \rho_s(\theta_{out}, \theta_{in}) I_{in}(\theta_{in}) \cos(\theta) d\omega$$

Pass 1 – Modified Radiosity

- Standard radiosity algorithm (hemi-cube)
 - Extend to account for translucency
 - Extend to account for specular-diffuse transport
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- For both extensions, introduce new form factors

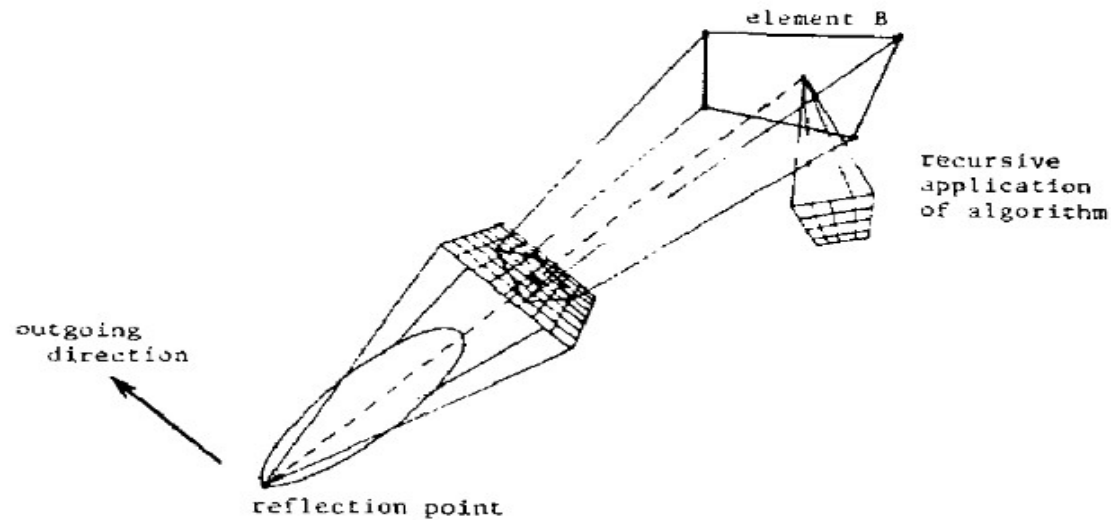
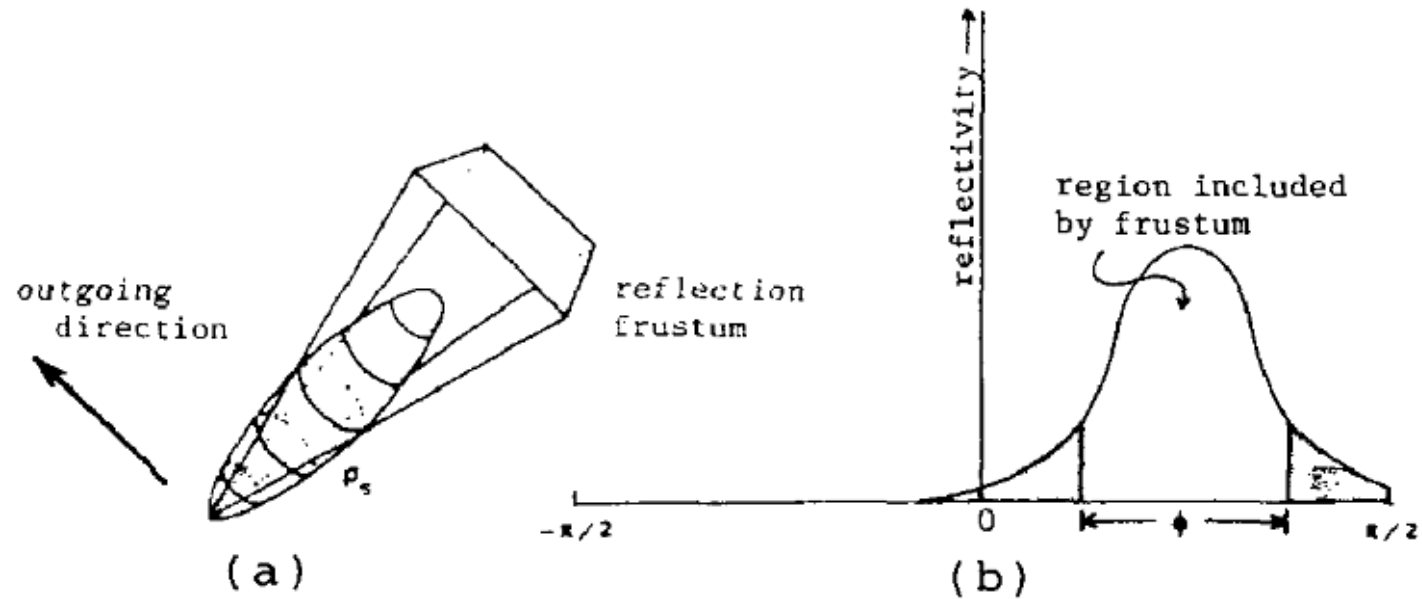
Pass 1 – Modified Radiosity



Pass 2 – Modified Ray Tracing

- Specular-specular reflection is captured well by classical ray tracing
- Extend ray tracing to take into account arriving diffused light as well
- Light arrives over entire hemisphere, but is only influential over small solid angle due to BRDF. This area can be discretized

Pass 2 – Modified Ray Tracing



All Terms Are Now Known

$$\rho''(\theta_{out}, \theta_{in}) = k_s \rho_s(\theta_{out}, \theta_{in}) + k_d \rho_d$$

where

$$\begin{aligned} k_s &= \text{fraction of reflectance that is specular} \\ k_d &= \text{fraction of reflectance that is diffuse} \\ k_s + k_d &= 1 \end{aligned}$$

$$I_{out}(\theta_{out}) = E(\theta_{out}) + I_{d,out} + I_{s,out}(\theta_{out})$$

$$I_{d,out} = k_d \rho_d \int I_{in}(\theta_{in}) \cos(\theta) d\omega$$

$$\begin{aligned} I_{s,out}(\theta_{out}) &= \\ & k_s \int \rho_s(\theta_{out}, \theta_{in}) I_{in}(\theta_{in}) \cos(\theta) d\omega \end{aligned}$$