Approximating Terrain with Over-determined Laplacian PDEs

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ABSTRACT

We extend Laplacian PDE by adding a new equation to form an over-determined system so that we can control the relative importance of smoothness and accuracy in the reconstructed surface. Benefits of the method include the ability to process isolated, scattered elevation points and the fact that reconstructed surface could generate local maxima, which is not possible in the original Laplacian PDE by the maximum principle. We use certain geometric algorithm including Triangulate Irregular Network, Visibility test, Level Set Component that discovers important points which reflect the terrain structure and use our extended Laplacian PDE to approximate the terrain from these points. We present experiments and measurements using different metrics and our method gives convincing results.

1. INTRODUCTION

Nowadays, the size of digital terrain data has grown to an extent that makes it essential to use some special representation or compression technique to manipulate the data. However, the development of processing and handling of digital terrain data has not advanced in pace with the data inflation. Elevation datasets are still stored as an elevation matrix. Common algorithms for compressing these matrices were originally designed for problems not specifically related to GIS and tend to yield poor results. For example, gzip, which USGS DEM data are usually compressed with, was originally designed as a plain text compressor.[1] In Table 1, we list the compressed size using gzip for an 'unfair' comparison (because gzip is lossless).

In this paper, we use Over-determined Laplacian Partial Differential Equations (ODETLAP) to approximate and lossily compress terrains. We construct an over-determined system using points selected by one of the four strategies: triangulation, visibility tests, level set components and random selection; then use an over-determined PDE to solve for a smooth approximation. After that, we refine the approximation with respect to the original terrain by adding into the important points set points with biggest elevation error or slope error, and then use ODETLAP to solve for a better approximation. These two steps are alternately applied until the error is satisfactorily small.

ODETLAP can process not only continuous contour lines but isolated points as well. The surface produced tends to be smoothe while preserving high accuracy to the known points. Local maxima are also well preserved. Alternative methods generally sub-sample contours due to limited processing capacity, or ignore isolated points.

2. OVER-DETERMINED LAPLACIAN APPROXIMATION OVERVIEW

Since we are working on single value terrestrial elevation matrix, We have the Laplacian equation

$$4z_{ij} = z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} \tag{1}$$

for every unknown non-border point. In terrain modeling this equation has the following limitations:

- The solution of Laplace's equation never has a relative maximum or minimum in the interior of the solution domain, this is called the "maximum principle";[2] so local maxima are never generated.
- The generated surface may droop if a set of nested contours is interpolated[3]

To avoid these limitations, an over-determined version of the Laplacian equation is defined as follows: apply the equation (1) to every non-border point, both known and unknown, and a new equation is added for a set S of known points:

$$z_{ij} = h_{ij} \tag{2}$$

where h_{ij} stands for the known elevations of points in S and z_{ij} is the "computed" elevation for every point, like in equation (1). The system of linear equations is over-determined, i.e., the number of equations exceeds

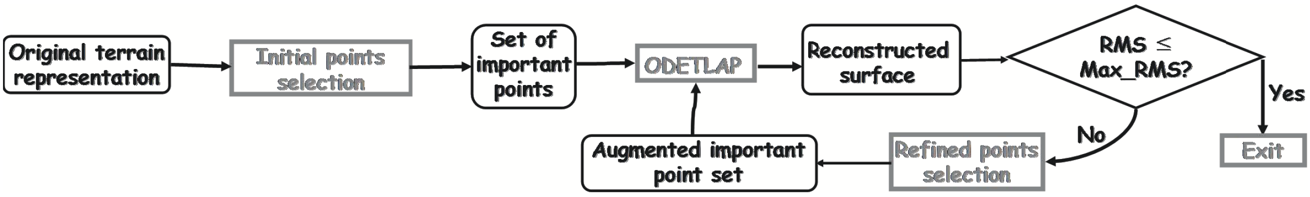

Figure 1. Algorithm Outline

Data	Elev. Range	RMS	Mean	Max	XY Size	Z Size	Total Size	Comp. Ratio	gziped size
Hill1	505.5	3.62	2.82	16.97	1446	1304	2750	116.4	113468
Hill2	745	9.45	7.32	40.52	1472	1354	2826	113.2	177633
Hill3	500	1.72	1.35	10.41	1400	1209	2609	122.7	74973
Mtn1	1040	17.34	13.47	76.80	1493	1456	2949	108.5	213217
Mtn2	953	17.17	13.48	65.22	1476	1424	2900	110.3	213537
Mtn3	788	17.06	13.36	87.52	1462	1503	2965	120.8	212093

Table 1. ODETLAP Compression results: Each of the ODETLAP tests consist of 100 initial points selected with the TIN method, and then 10 points are added using the greedy selection method on each iteration for 90 iterations, for a total of 1000 points.

the number of unknown variables, so instead of solving it for an exact solution (which is now impossible), an approximated solution is obtained by setting up a smoothness parameter R that determines the relative importance of accuracy versus smoothness.

3. ALGORITHM OUTLINE

The ODETLAP algorithm's outline is shown in figure 1. Starting with the original terrain elevation matrix there are two point selection phases: firstly, the initial point set S is built and a first approximation is computed using the equations (1) and (2). Given the reconstructed surface, a stopping condition based on an error measure is tested. If this condition is not satisfied, the second step is executed. In this step, $k \geq 1$ points from the original terrain are selected according to the error in the reconstructed surface and are inserted in the existing point set S; this extended set is used by ODETLAP to compute a more refined approximation. As the algorithm proceeds, the total size of point set S increases and the total error converges.

4. RESULT AND ANALYSIS

We test our algorithm on various real world terrain data sets. Each data set is a 400×400 elevation matrix and original binary size is 320KB. In Table 1 we have results showing that our new compression scheme gets compression ratio of over 100 and the mean absolute error is no more than 2% in all cases. We use TIN to find the initial 100 important points and select 10 points in each of the 90 iteration, so at the end, the number of points we need to save is 1000.

5. FUTURE WORK

The next step of research consists of a few extensions in two directions: one is higher accuracy. We will investigate other PDEs to see if they can reconstruct the terrain more accurately than the Laplacian PDE. Another direction is higher compression. Currently we use lossless compression in the final compression step. We will test the use of lossy schemes, which can reach higher compression ratio at the cost of accuracy. Since slope is also a very important feature of terrain, we will also consider ways to minimize slope error in our representation.

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