# Segmented Height Field and Smoothed Particle Hydrodynamics in Erosion Simulation

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## 1 Motivation

The New Orleans area levee failures during Hurricane Katrina drew media attention to an important problem in Civil Engineering. The emphasis of our work is on earthen levees, dams, and embankments. A major cause of failures of such structures is overtopping, which causes erosion to the point of breaching the crest. Our research focuses on simulating the initial small-scale features of erosion – the formation of rills and gullies on the embankment. We wish to study and eventually be able to simulate the way earthen embankments erode, with respect to the formation of these rills and gullies. Validation of computer simulations is the primary focus of our research. We will utilize RPI's geotechnical centrifuge to perform high-g erosion experiments on small-scale models to predict and validate the model for full scale simulations.

## 2 Review of Literature

**Erosion Models and Erodibility** A variety of existing erosion models calculate the overall soil loss during the overtopping of an earthen embankment. For example, Wang and Kahawita present a two-dimensional mathematical model of erosion of the profile of an earthen embankment during overtopping (Wang and Kahawita 2003). The "erodibility" of soil is generally defined as the ratio of the rate the soil erodes to the velocity of the water causing the erosion. In his work, Jean-Louis Briaud defines erodibility as a function of hydraulic shear stress, or pull of the water on the soil and presents measurements of soil samples collected from many of the affected earthen levees in the New Orleans area (Briaud, Chen, Govindasamy, and Storesund 2008).

**Erosion Simulations** Kristof et al., present an erosion simulation using smoothed particle hydrodynamics (SPH). The soil, water, and soil-water boundaries are represented by particles – the soil and water particles have mass and velocity while the boundary particles are designed solely for the two phases to interact (Kristof, Benes, Krivanek, and St'ava 2009).

**Data Structures** Three data structures dominate erosion research in computer graphics: the height field (or height map), the layered height field, and the voxel grid (see Figure 2). Height fields are a regular 2D grid array of heights, where each grid space allows for a single height value. Height fields, like the one used in (Benes and Arriaga 2005), are easily compressible, and allow for fairly simple surface extraction. However, they do not allow for layered terrains.

A voxel based terrain representation allows for immediate coupling with many fluid simulation techniques. Voxel-based terrain representations allow for multiple soil layers, different soil parameters, caves and undercuts, and inherent volume information. Tetrahedral meshes are another volumetric representation that builds the model out of tetrahedra. This representation uses the points along the surface of the volume, and thus is more accurate in representing the model's surface than a voxel grid, which is limited to the grid resolution in three dimensions.

Benes et al. present a layered height field that combined several advantages of each of height fields and voxel grids (Benes and Forsbach 2001). The terrain is divided into a two dimensional grid, like a height field, but each grid space contains an array of heights. This representation allows for several different soil types, a surface can be easily extracted for visualization and simulation purposes, the precision is arbitrary, and caves and undercuts are possible.

## 3 Segmented Height Field

Our new data structure is the Segmented Height Field (SHF), shown in Figures 1&2d. It is similar to the layered height field, described earlier, in that it is a grid of cells, and each cell contains a list of the layers of soil found in that column. In our structure, each cell (referred to as a **column**) contains a list of soil **segments** (a rectilinear block of soil). Segments of the same soil type in adjacent grid columns that have overlapping intervals in height create a **layer**.

Several important differences exist between the layered height field and SHF. SHF allows for layers to be dynamically added and removed, making accurate re-deposition easier. Second, a layered height field does not allow for two layers of the same soil

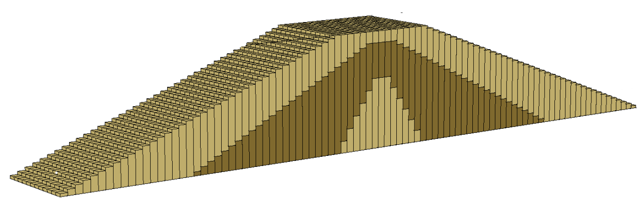
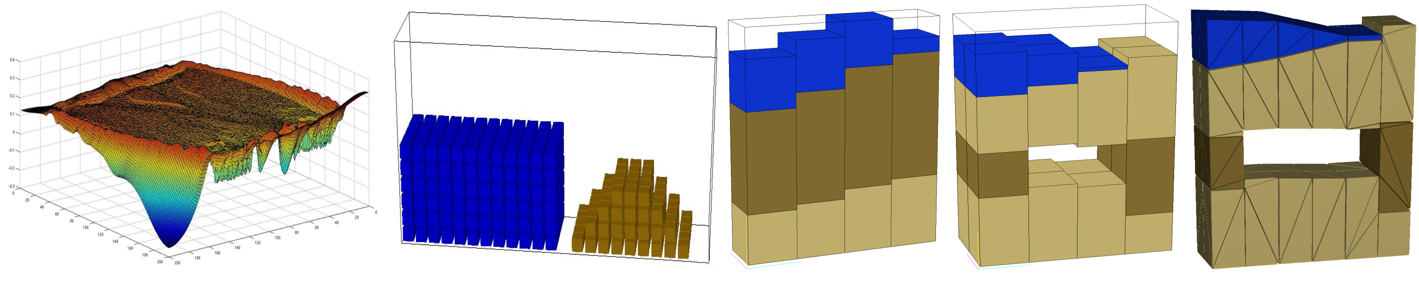


Figure 1: Earthen embankment with multiple soils.



**a)** **b)** **c)** **d)** **e)**

Figure 2: Surface and Volumetric Data Structures: a) single-valued height field, b) voxel grid, c) layered height field d) our new Segmented Height Field (SHF), e) interpolated tetrahedral mesh based on the SHF.

type to exist in one list in one grid cell, whereas in our representation these layers are added automatically. Because there are no restrictions with regards to the location of a soil segment within the column SHF naturally supports overhangs and air pockets.

Furthermore, each SHF segment can have spatially varying soil parameters; for example, moisture content. Yet our SHF retains a key advantage of height fields: the layer height is not limited in resolution as it is for a voxel grid. Finally, the SHF can be more memory efficient than either a voxel grid or a layered height field. Voxel grids store redundant data when representing layers that are thick relative to the underlying grid. And layered height fields are inefficient for accurate representation of complex soil re-deposition, because the addition of new layers is global rather than local.

## 4 Tetrahedralization

For visualization and simulation purposes, it is necessary to be able to generate an interpolated volume from our SHF. To do this, we construct, from the segmented height field, a tetrahedral mesh that explicitly captures the slope and surface normals of the boundary surfaces between air and soil and between different soil types. These not only improve the quality of the resulting visualization, but also will yield more accurate physical simulations by allowing water to flow smoothly down the inclined slope of the embankment and through channels cut within the soil.

To convert the SHF to a tetrahedral mesh, we define a set of rules to govern the placement of pivot points. The points are either placed at T-junctions in the data or interpolated along layer surfaces. The points are then connected in a manner that ensures the formation of a closed, water-tight tetrahedral mesh. The resulting volume has a smooth, water-tight surface on which erosion simulations may be run.

## 5 Smoothed Particle Hydrodynamics

The simulation of water and its erosion on the levee will be based on Smoothed Particle Hydrodynamics (SPH). In the SPH method, the state of a system is represented by a set of particles, which possess individual material properties and move according to some governing conservation equations. The most significant advantage of this mesh-free method lies in its adaptability and its ability of handling problems of extremely large deformation. Unlike the widely used voxel-based fluid simulation methods, the mess-free SPH method is better suited to adapt to changing soil-fluid boundary conditions. Therefore, we believe it is most suitable for simulating levee erosion. In the simulation, water particles and soil particles will first be handled independently based on SPH theories, and then coupled to simulate the interaction between water and soil.

## 6 Results and Future Work

To date, we have collected LIDAR scan data of rill and gulley formation on a small earthen embankment and loaded it into our new SHF data structure. In the future, we will be able to simulate water flowing over the embankment, using SPH, and eroding the soil away to form rills and gulleys. We will then perform high-g erosion experiments on RPI's centrifuge in order to statistically validate our simulation results. This research will lead to new software tools for civil engineers that help analyze and predict the performance of earthen embankments.

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