

Blind, Greedy, and Random: Algorithms for Matching and Clustering Using only Ordinal Information

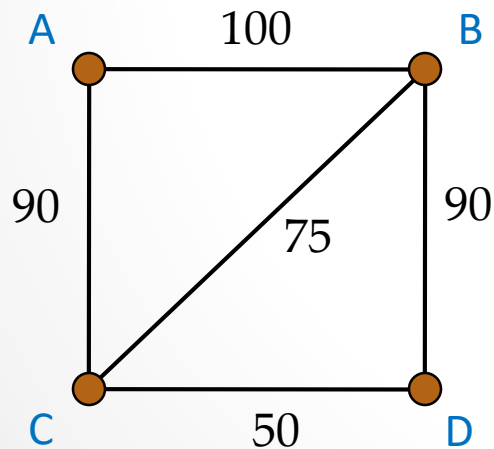
Elliot Anshelevich

(together with Shreyas Sekar)

Rensselaer Polytechnic Institute (RPI), Troy, NY

Maximum Utility Matching

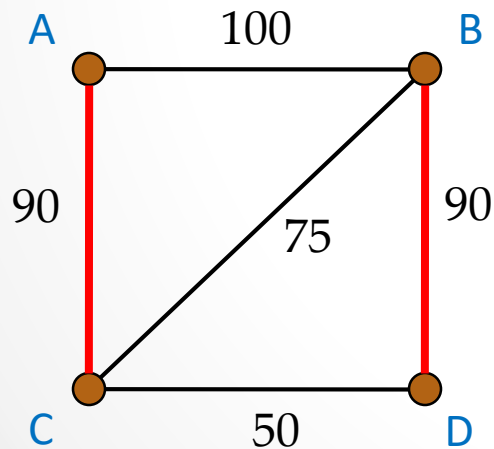
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- For example, weight can represent compatibility, utility from matching this pair



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Goal: maximize social welfare = total utility

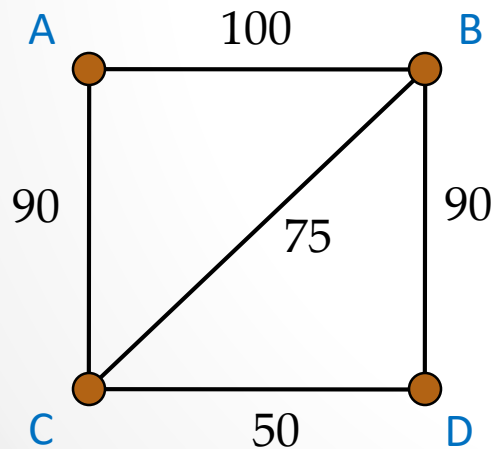


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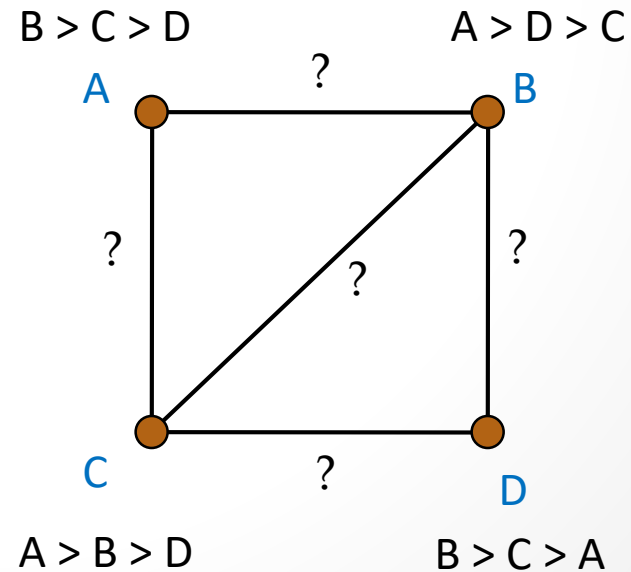
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What if we only know **ordinal preference information**?

Truth



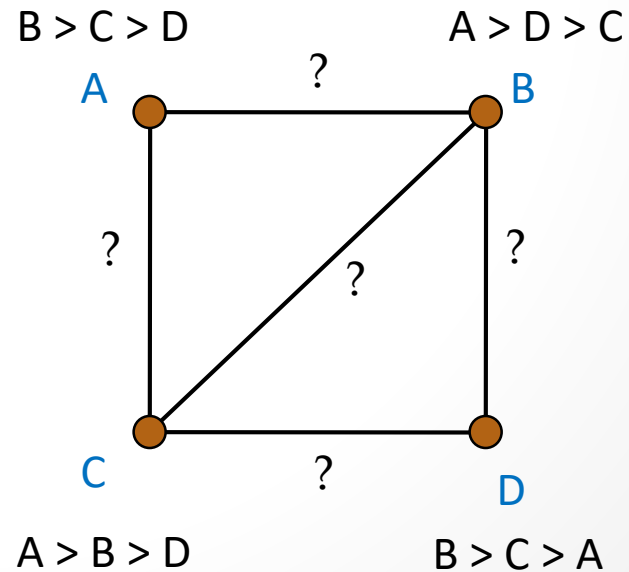
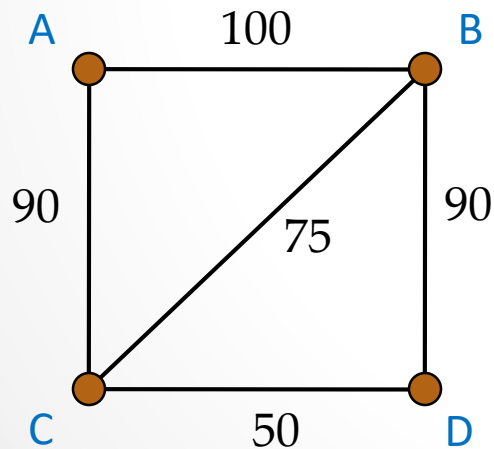
What we know



Ordinal Approximations

What if we only know ordinal preference information?

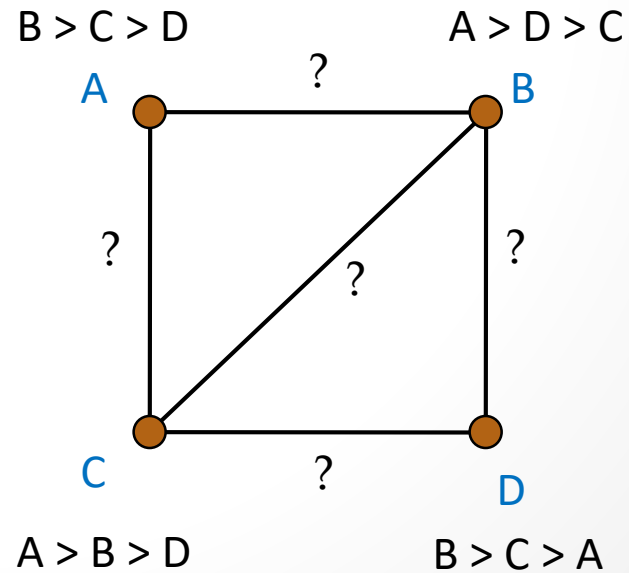
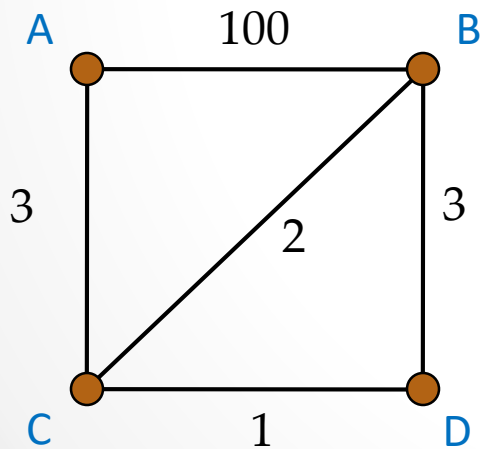
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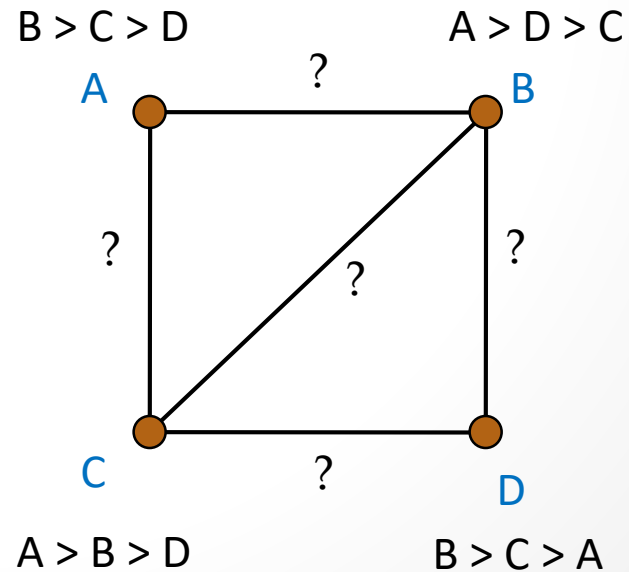
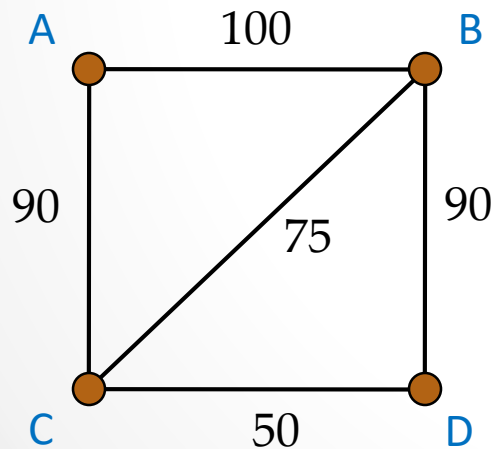


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What if we only know ordinal preference information?

Goal: ~~Compute max-utility matching using only ordinal information.~~

Approximate max-utility matching using only ordinal information.

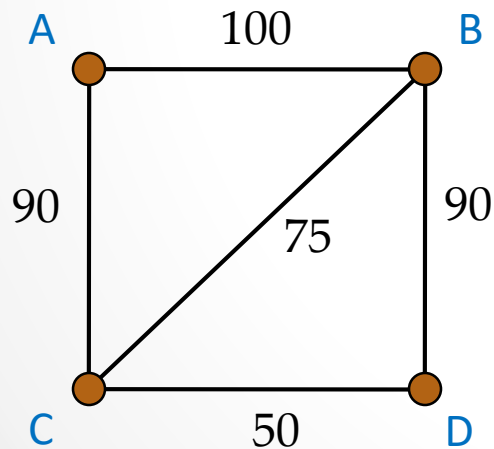


Ordinal Approximations

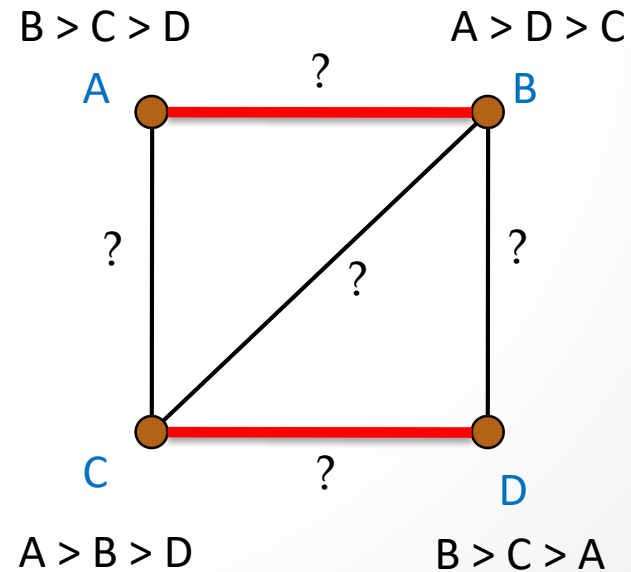
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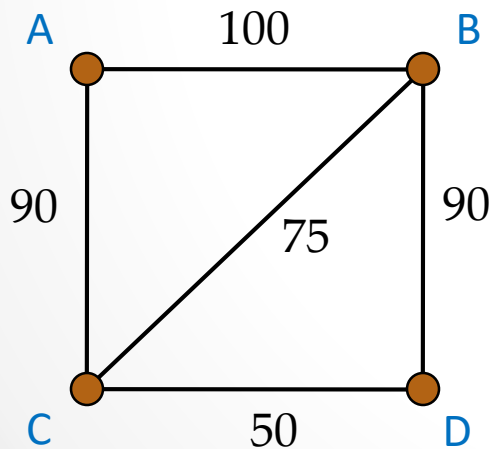


Greedy Algorithm

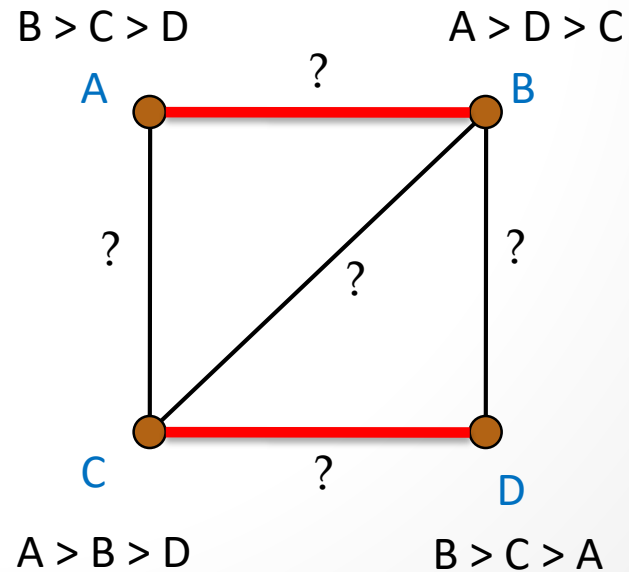
- Pick edge (X, Y) of maximum weight.
- Remove X and Y , and repeat.

Classic algorithm; produces 2-approximation.

Truth



What we know

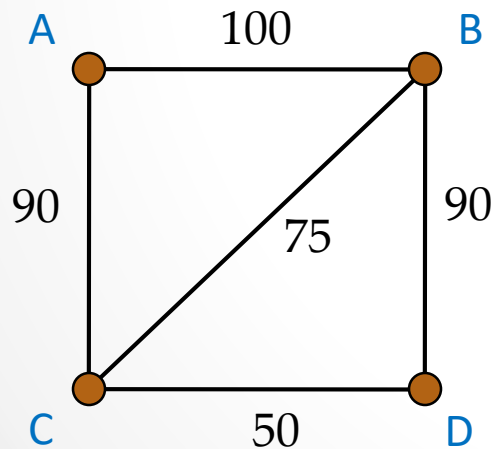


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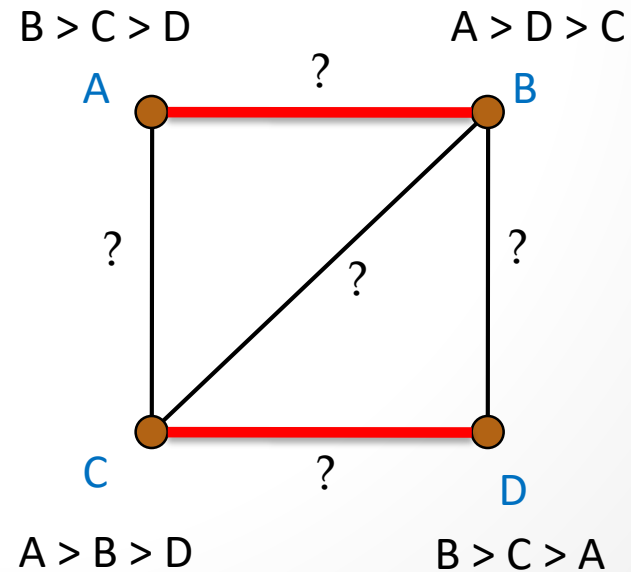
- Pick edge (X, Y) such that X is Y 's first choice, and Y is X 's first choice.
- Remove X and Y , and repeat.

Classic algorithm; produces 2-approximation *no matter what the true weights are!*

Truth

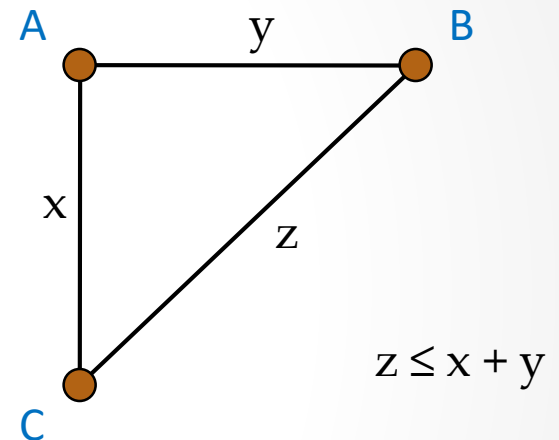


What we know



Ordinal Approximation for Metric

Can we do better? Will look at *metric* weights, i.e., weights that obey triangle inequality.

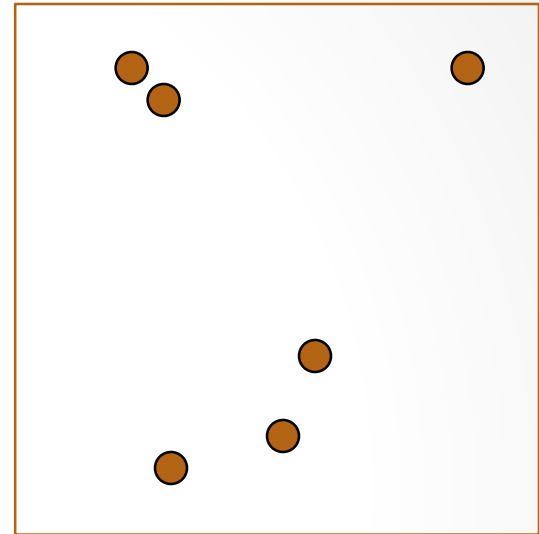


Will provide a

- 1.6-ordinal approximation
(nothing better than 1.25 is possible)
- Framework for ordinal approximations:
useful for clustering problems, traveling salesman, etc.

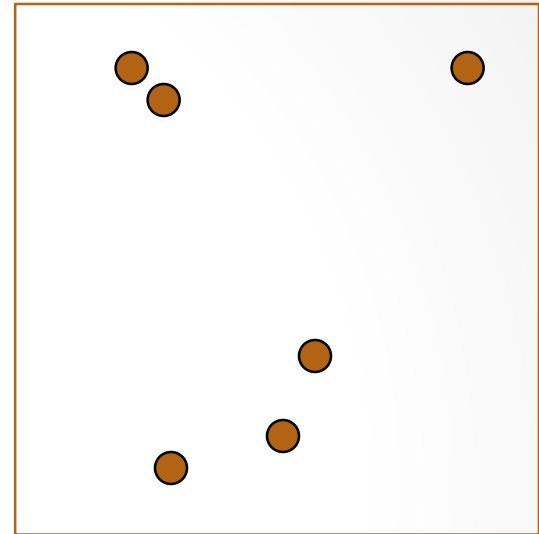
Maximum Weight Metric Matching

- Diverse Team Formation
 - Want partners with complementary skills
 - Matching is teams of two

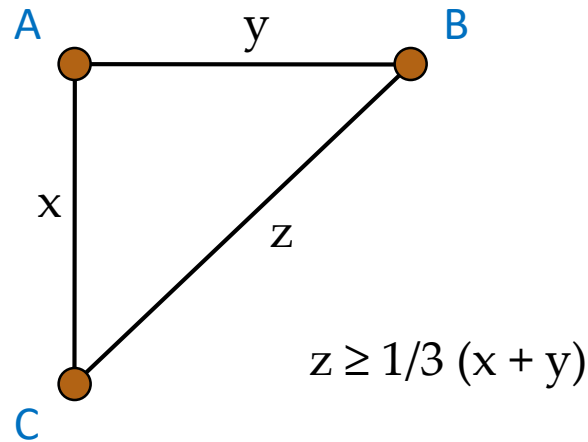


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- Homophily

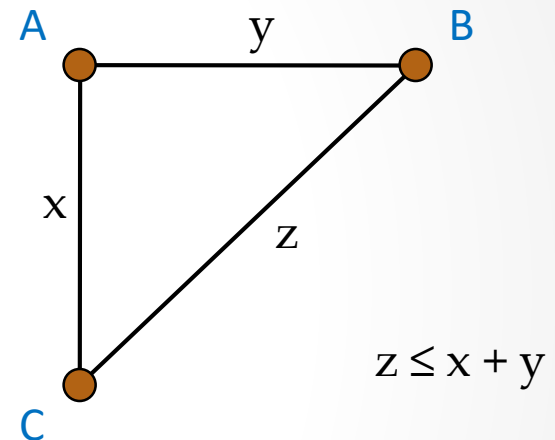


Ordinal Approximation for Metric

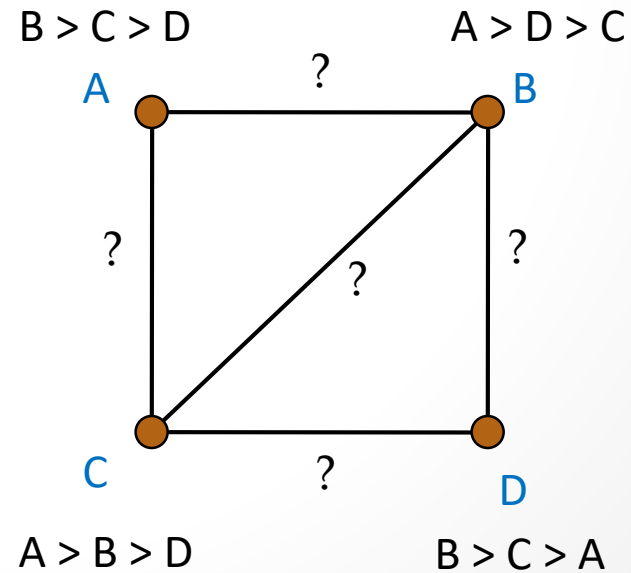
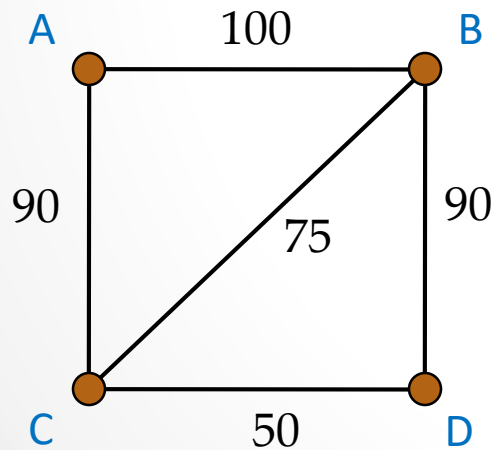
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Random: Pick a random matching

For metric weights:
produces 2-approximation to maximum-weight matching!

- Can we take better of two algorithms? Don't even know what "better" is!
- Can we mix over two solutions? Yes, but can do even better.

1.6-approximation to Max Weight Matching

- Run Greedy until match $2/3$ of the nodes



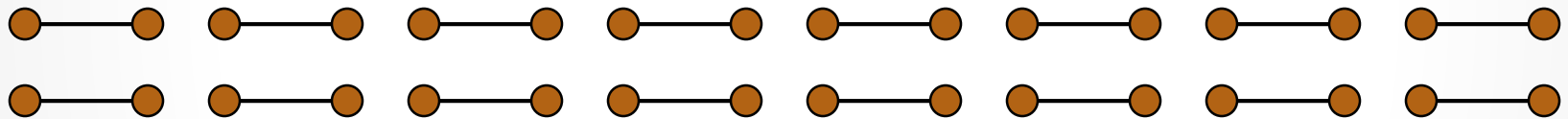
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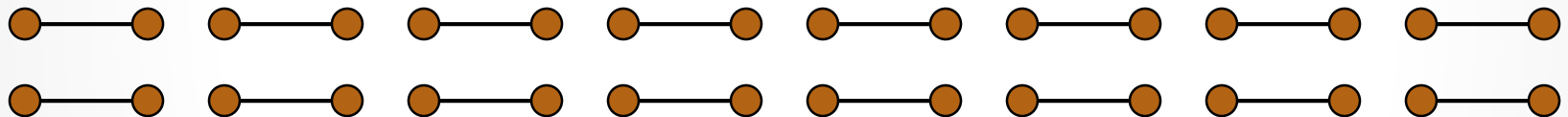
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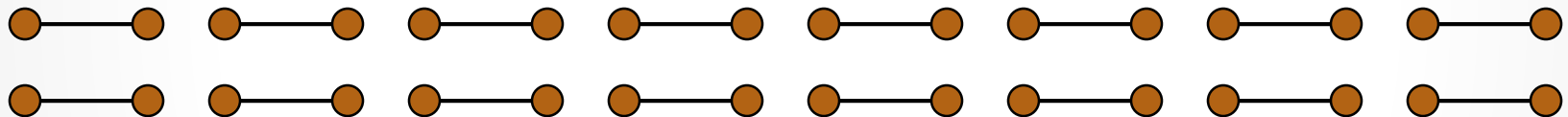
- Run Greedy until match 2/3 of the nodes



Claim: Top half of edges in Greedy Matching are already 2-approx to Max-Weight Matching.

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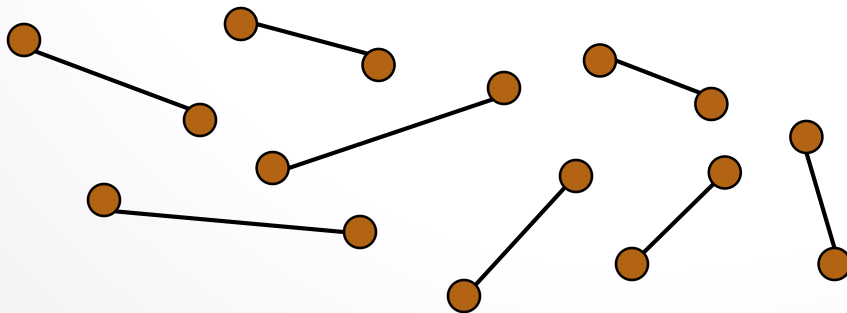
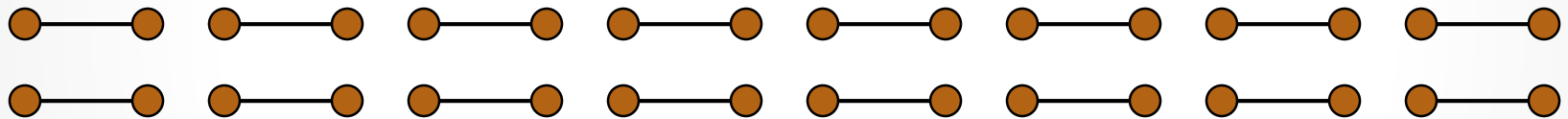


Claim: Top half of edges in Greedy Matching are already 2-approx to Max-Weight Matching.

Claim: Running Greedy until 2/3 of nodes are matched is a 2-approx.

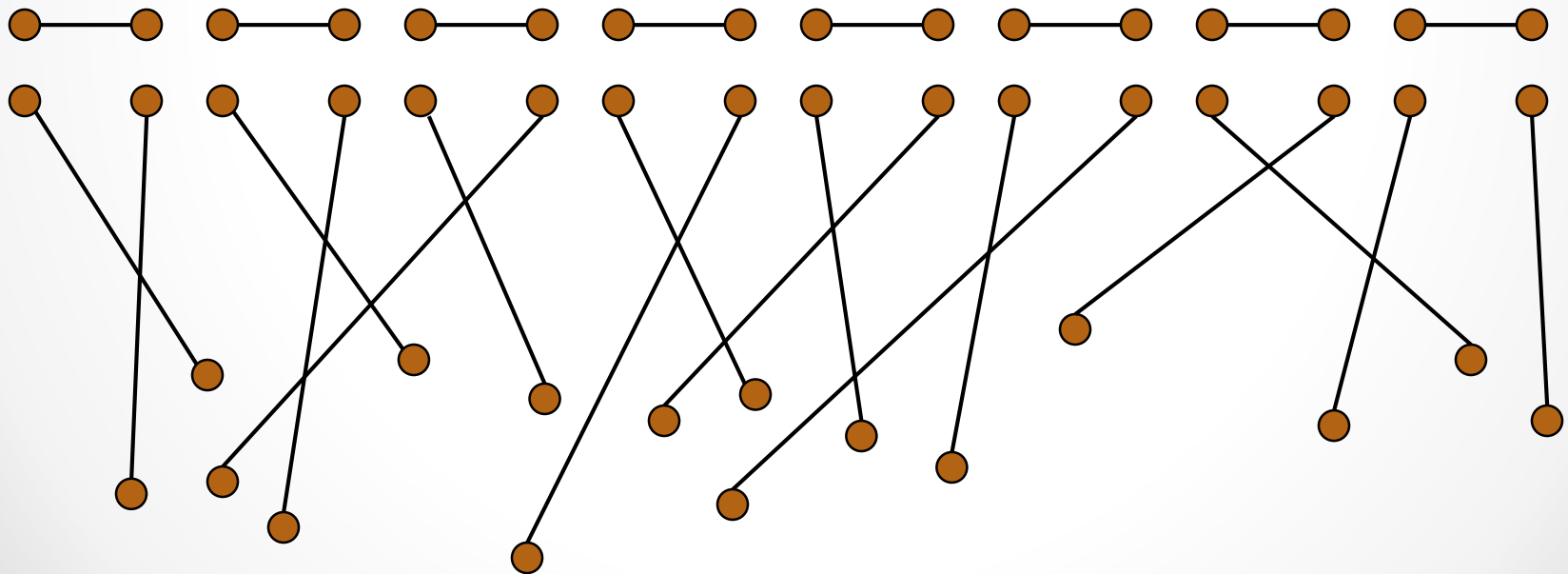
1.6-approximation to Max Weight Matching

- Run Greedy until match 2/3 of the nodes
- Solution 1: Form random matching on rest of nodes



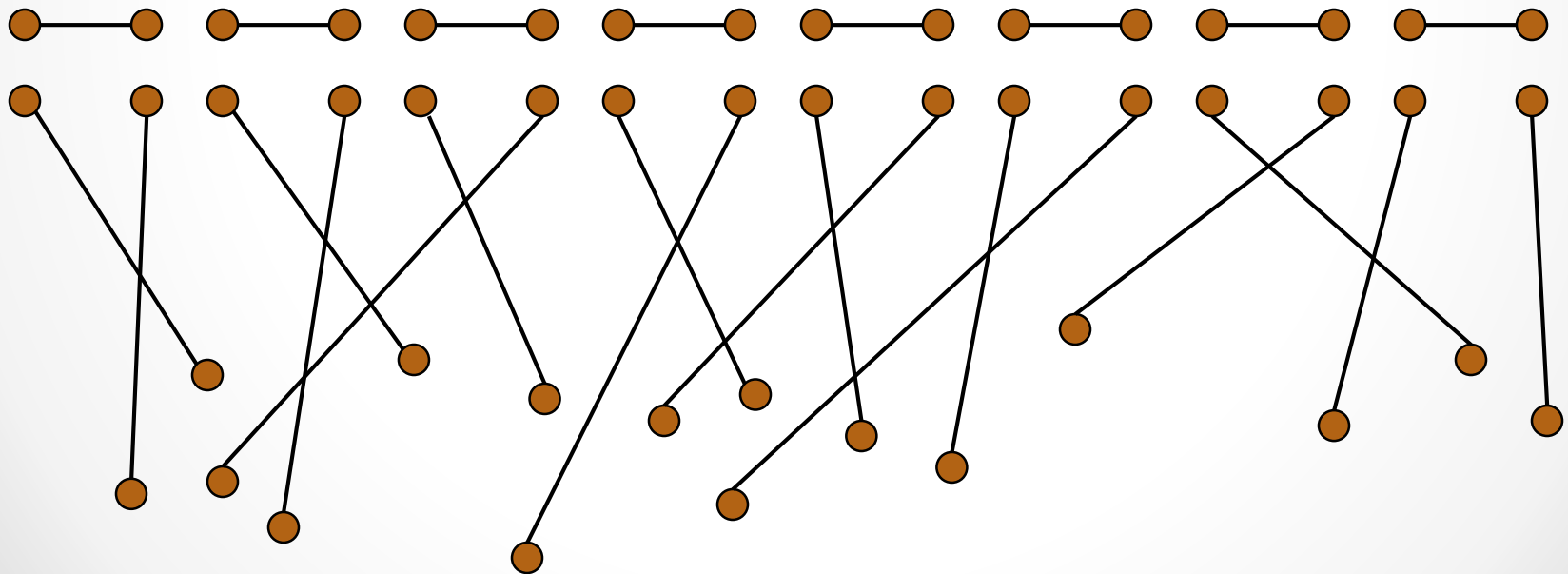
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- Run Greedy until match $2/3$ of the nodes
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- Solution 2: Form random bipartite matching to rest of nodes

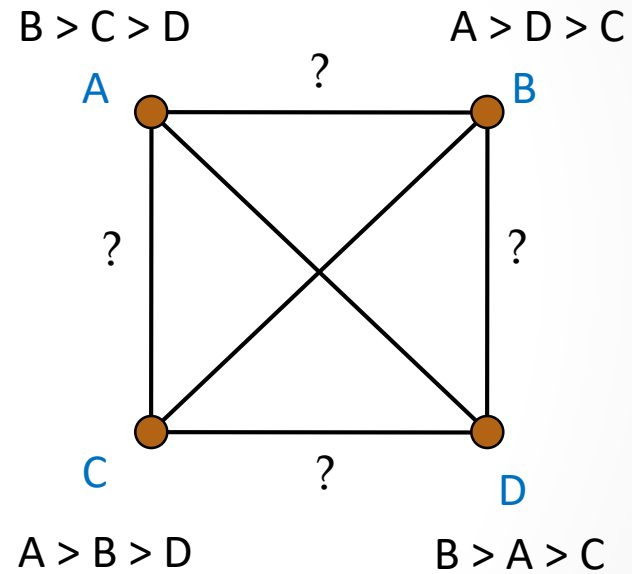
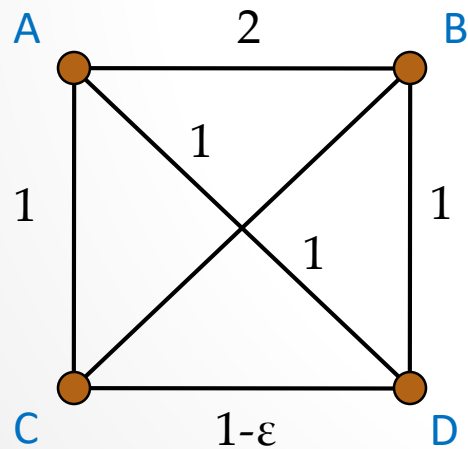
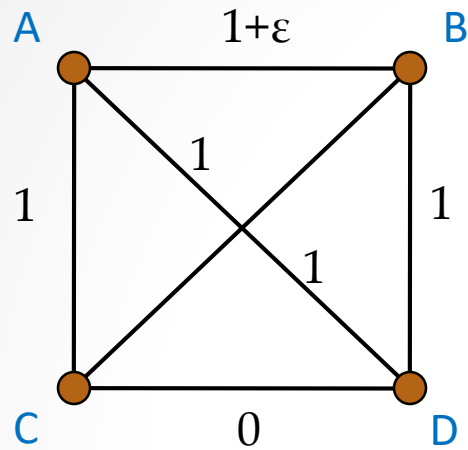


1.6-approximation to Max Weight Matching

- Run Greedy until match $2/3$ of the nodes
- Solution 1: Form random matching on rest of nodes
- Solution 2: Form random bipartite matching to rest of nodes
- Take each solution with probability $1/2$

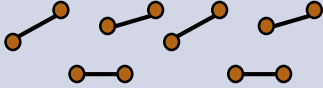
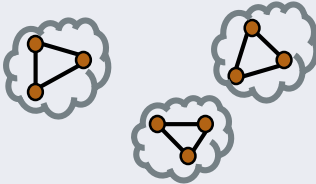
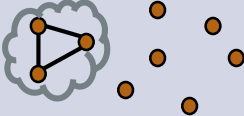
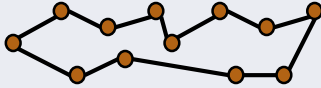


Lower Bound Example



OPT/E[any alg] is
no better than 1.25

Ordinal Approximations Using this as a Black Box

	Full Information		Ordinal Approximation
Maximum Weight Matching	1		1.6
Max k-sum clustering	2		2
Densest k-subgraph	2		4
Max Metric Traveling Salesman (TSP)	1.14		1.88

Ordinal Approximations Using this as a Black Box

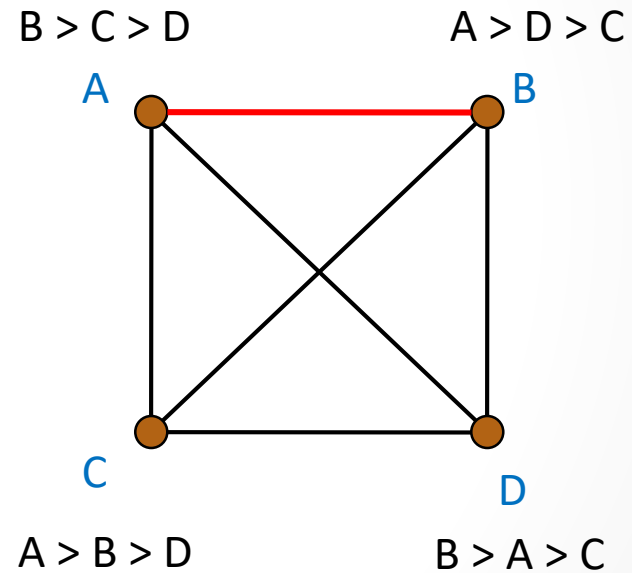
	Full Information	Black Box Reduction	Ordinal Approximation
Maximum Weight Matching	1	α	1.6
Max k-sum clustering	2	2α	2
Densest k-subgraph	2	$2(\alpha \text{ for k-matching})$	4
Max Metric Traveling Salesman (TSP)	1.14	$4\alpha/3$	1.88

Ordinal Approximations Using this as a Black Box

	Full Information	Black Box Reduction	Ordinal Approximation
Maximum Weight Matching	1	1.6	1.6
Max k-sum clustering	2	3.2	2
Densest k-subgraph	2	4	4
Max Metric Traveling Salesman (TSP)	1.14	2.14	1.88

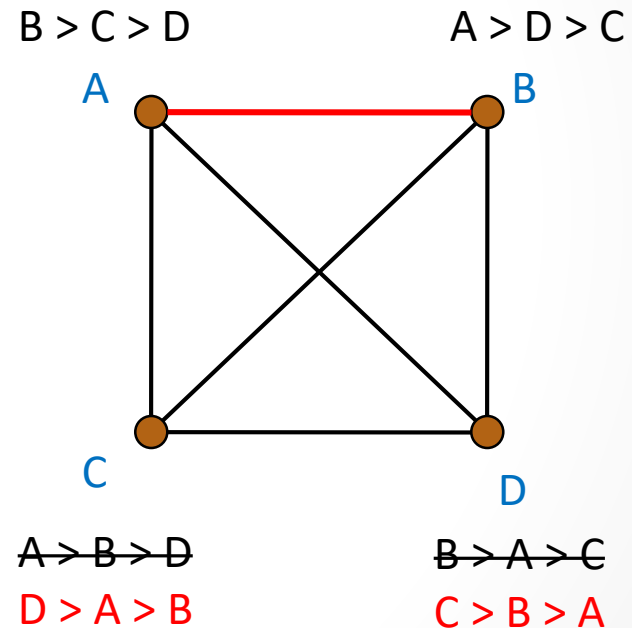
Truthful Matching

- Running Greedy to form perfect matching is truthful
- Running Greedy to form **k-matching** is not truthful



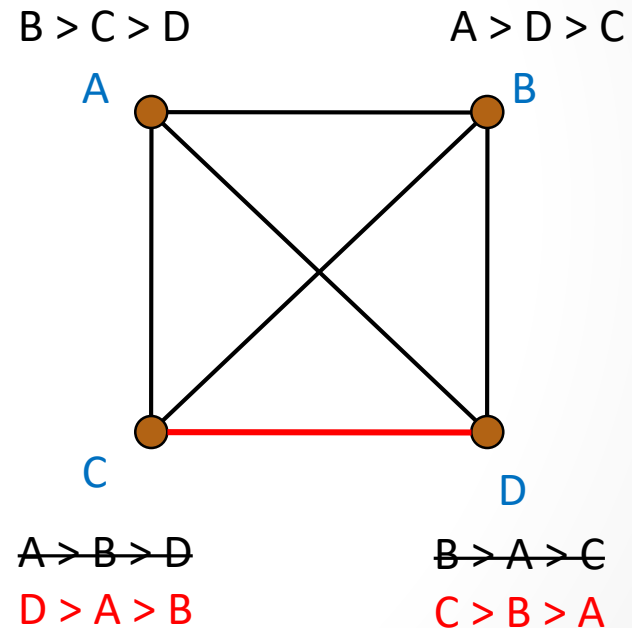
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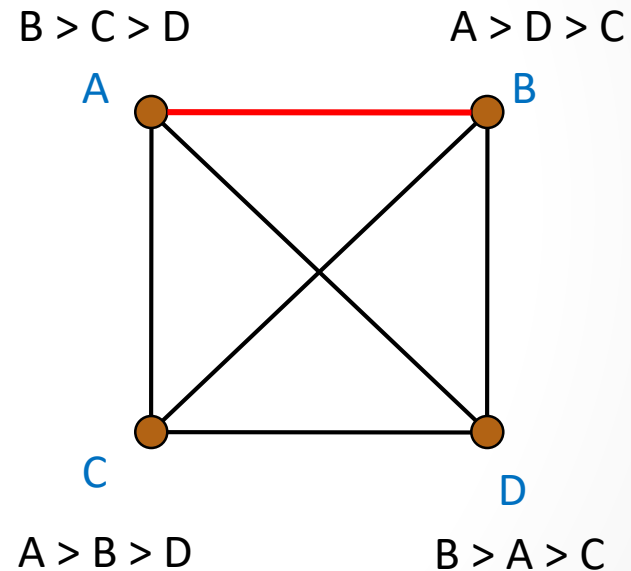
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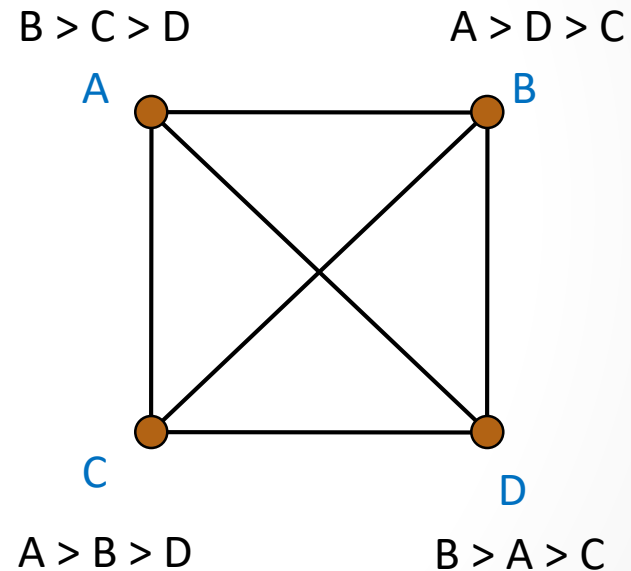


- Instead can use *Random Serial Dictatorship*: 2-approximation

Truthful Matching

- Running Greedy to form perfect matching is truthful
- Running Greedy to form **k-matching** is not truthful

Take top preference of random node
Remove these nodes from graph
Repeat



- Instead can use *Random Serial Dictatorship*: 2-approximation

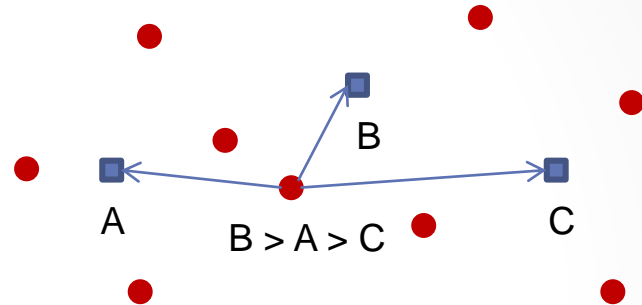
Ordinal Approximations Using this as a Black Box

	Full Information	Truthful Ordinal Approximation	Improved (non black-box)
Maximum Weight Matching	1	1.76	1.6
Max k-sum clustering	2	2	2
Densest k-subgraph	2	6	4
Max Metric Traveling Salesman (TSP)	1.14	2	1.88

Other Ordinal Problems

- Ordinal problems in social choice

- Facility location



- Min-cost matching, Minimum Spanning Trees

- Non-metric shortest path vs longest tour