FINAL: 180 Minutes

Last Name: _____

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

1-12	Total
10 each	120

1 **Determine the type of proof and prove: a right triangle with integer sides cannot be isoceles.**

2 **Prove that the product of any 5 consecutive natural numbers is divisible by $5!$.**

3 **Let $G_0 = 0$, $G_1 = 1$, and $G_n = 7G_{n-1} - 12G_{n-2}$ for $n > 1$. Compute G_5. Show $G_n = 4^n - 3^n$ for $n \geq 0$.**

4 **Determine and prove the order-relationships between $\ln n$, $\ln(n^2+1)$, and $\ln(2n)$.**

5 **Prove that**

$$xk \equiv yk \mod d$$

implies that

$$x \equiv y \mod (d/\gcd(k, d)).$$

6 **Adam, Barb, Charlie, and Doris each independently choose a random number uniformly distributed in $\{1, 2, 3, 4, 5\}$. What is the probability that some pair chooses the same number? What if there are k people and n numbers?**

7 **Let $\mathcal{V}$ be a set of n vertices, and let the edge set $\mathcal{E}$ be initially empty. For each pair of vertices $i \neq j$, add the edge (i, j) to $\mathcal{E}$ with probability p. Give the pdf for the degrees of the nodes of this graph.**

8 **Voltage in the US has a mean of 120V and a standard deviation of 5V. A device's operating voltage is 112–128. Use Chebyshev's inequality to bound the probability that the device will not be damaged when turned on.**

9 Give a DFA for strings whose even digits alternate between 0 and 1.

10 **For CFG $S \to 0S|S1|0|1$, prove that no string has 10 as a substring.**

11 **Consider the language of palindromes $\mathcal{L} = \{\omega \,|\, \omega \in \{0,1\}^*, \omega = \omega^R\}$. Give well-written high-level pseudocode for a decider for this language.**

12 **Given an ultimate-debugger $\mathcal{D}$ that takes $\langle M \rangle \# \omega$ and decides if TM M halts on input ω, show that every recognizer of a language $\mathcal{L}$ can be converted into a decider for the language.**

SCRATCH

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