

CSCI 6220/4030: Homework 5

Assigned Tuesday November 13 2017. Due at beginning of class Monday November 26 2017.

Remember to typeset your submission, and label it with your name. Please start early so you have ample time to see me during office hours. Provide mathematically convincing arguments for the following problems. Ask me if you are unclear whether your arguments are acceptable.

1. Two rooted trees T_1 and T_2 are said to be isomorphic if there exists a one-to-one mapping f from the vertices of T_1 to those of T_2 satisfying the following condition: for each internal vertex v of T_1 with the children v_1, \dots, v_k , the vertex $f(v)$ has as children exactly the vertices $f(v_1), \dots, f(v_k)$. Observe that no ordering is assumed on the children of any internal vertex. Devise an efficient randomized algorithm for testing the isomorphism of rooted trees using the Schwartz-Zippel theorem, and analyze its cost and probability of success. Hint: associate a polynomial P_v with each vertex v in a tree T . The polynomials are defined recursively, with the base case being that the leaf vertices all have $P = x_0$. An internal vertex v of height h with the children v_1, \dots, v_k has its polynomial defined to be $(x_h - P_{v_1}) \cdots (x_h - P_{v_k})$. Note that there is one indeterminate x_h associated with each level h in the tree.
2. Consider two computers, each containing n bit strings of length n . It can be shown that any deterministic algorithm for determining whether these sets have a non-empty intersection requires $O(n^2)$ bits to be communicated between the computers. Design a Las Vegas algorithm for answering this problem that communicates $O(n \log n)$ bits in expectation *when the sets do not intersect*.