Consider the problem
\[
\text{argmin}_{x \in \mathbb{R}} \frac{1}{2} (x - a)^2 + \lambda |x|,
\]
where \( \lambda > 0 \) is a nonnegative constant. This is a simple example of ordinary least squares with \( \ell_1 \)-regularization.

(1) Argue that this is a convex optimization problem, and it has a unique solution, given any \( a \). Use our rules for constructing convex functions from simpler ones.

(2) Let \( s_\lambda(a) \) be the unique solution to this optimization problem, given an \( a \). State Fermat’s optimality condition as concisely as you can, using our rules for subdifferential manipulation.

(3) Use Fermat’s optimality condition to find an expression for \( s_\lambda(a) \), and draw a cartoon/plot of \( s_\lambda \) as I might in class.