MIDTERM: 120 Minutes

Answer **ALL** questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.
You **MUST** show **CORRECT** work, **even on multiple choice questions**, to get credit.

GOOD LUCK!

1	**2**	**3**	**4**	**5**	**6**	**Total**

100	**20**	**20**	**20**	**20**	**20**	**200**

1 Circle one answer per question. 10 points for each correct answer.

(a) Compute the sum $\sum_{n=1}^{4} 3^n$.

A 120.

B 121.

C 242.

D 243.

E None of the above.

(b) What is the last digit of 3^{11}?

A 1.

B 3.

C 7.

D 9.

E None of the above.

(c) A graph has degree sequence $[6, 6, 3, 3, 3, 2, 2]$. How many edges does this graph have?

A 12.

B 25.

C 30.

D Not enough information to say.

E Such a graph does not exist.

(d) Suppose a connected planar graph has 18 vertices, each of degree 3. Into how many regions does any planar representation of this graph split the plane?

A 6.

B 11.

C 27.

D 40.

E None of the above.

(e) Compute $102^{1211} \bmod 5$.

A 0

B 1

C 2

D 3

E 4

(f) Which of the following numbers evenly divides $102^{1211} - 3^{1211}$?

A 5

B 17

C 2

D 99

E None of the above

(g) The negation of "If Lassie vomits then she ate grass or she is sick" is:

A If Lassie didn't eat grass and is healthy, she will not vomit.

B Lassie vomited and did not eat grass and is not sick.

C When Lassie eats grass or is sick, she does not vomit.

D Lassie did not vomit and she ate grass and is sick.

E None of the above.

(h) Which claim below is true?

A If $x, y \in \mathbb{Q}$ then $y^x \in \mathbb{Q}$.

B x is odd if and only if $x^2 - 1$ is divisible by 8.

C If p is prime, then $k^p - k$ is not divisible by p, for any integer k.

D None of these claims are true.

E All of these claims are true.

(i) Which of the following asymptotic relationships is correct?

A $(n+1)! \in O(n!)$.

B $(n+1)! \in \omega(n!)$.

C $(n+1)! \in o(n!)$.

D $(n+1)! \in \Theta(n!)$.

E None of the above.

(j) Which of the following recursions defines a sequence T_n satisfying $T_n \in \Theta(2^n)$?

A $T_1 = 2; T_n = T_{n-1}^2$ for $n > 1$.

B $T_1 = 2; T_n = 2 + 2T_{n-1}$ for $n > 1$.

C $T_1 = 2; T_n = 2nT_{n-1}$ for $n > 1$.

D All of the above.

E None of the above.

2 **Let p be prime. Consider an integer $b \in [1, p-1]$. Use Bezout's Theorem to show that there exists an integer $x \in [1, p-1]$ that satisfies $bx \equiv 1 \bmod p$.**

3 Prove or disprove: every graph with n vertices and $n-1$ edges is a tree.

4 For any positive integer k, prove that $1^k + 2^k + \cdots + n^k \in \Theta(n^{k+1})$.

5 **Let** $A_n = \underbrace{1 \cdots 1}_{n \text{ ones}}$ **for** $n \geq 1$**. Notice that** $A_n = 10A_{n-1} + 1$ **for** $n \geq 2$**. Use induction to show that** $A_n \equiv 3 \bmod 4$ **when** $n \geq 2$**.**

6 **Determine the type of proof, and prove: every odd natural number is the difference of two perfect squares.**

SCRATCH

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