MIDTERM: 120 Minutes

Last Name: ______________________

First Name: ______________________

RIN: ______________________

Section: ______________________

Answer **ALL** questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.
You **MUST** show **CORRECT** work, **even on multiple choice questions**, to get credit.

GOOD LUCK!

1	2	3	4	5	6	Total

100	**20**	**20**	**20**	**20**	**20**	**200**

1 **Circle one answer per question.** 10 **points for each correct answer.**

(a) Compute the sum $\sum_{n=1}^{4} 5^{n+1}$.

A 3900.

B 3901.

C 3905.

D 3906.

E None of the above.

(b) Compute 20^{20} mod 7

A 1.

B 3.

C 4.

D 6.

E None of the above.

(c) A graph has degree sequence $[6, 6, 4, 3, 3, 2, 2]$. How many edges does this graph have?

A 13.

B 25.

C 30.

D Not enough information to say.

E Such a graph does not exist.

(d) Suppose a connected planar graph has 4 vertices and splits the plane into 3 regions. Which of the following are possible degree sequences for the graph?

A [2 2 2 2].

B [3 3 3 3].

C [3 3 2 2].

D None of the above.

E No such graph exists.

(e) What is the last digit of 103^{192}.

A 0

B 1

C 2

D 3

E 4

(f) Which of the following numbers evenly divides $5^{69} - 1$?

A 4

B 5

C 11

D 23

E None of the above

(g) The converse of "If induction is appropriate then the base case is true and the inductive step holds" is:

A If the base case is false and the inductive step is false, then induction is not appropriate.

B If the base case is false or the inductive step does not hold, then induction is not appropriate.

C If induction is not appropriate, then the base case is false or the inductive step does not hold.

D If the base case is true and the inductive step holds then induction is appropriate.

E None of the above.

(h) Which claim below is *not* true?

A $2n^2 + n \in \Theta(n^2)$.

B $4^n \in \Theta(2^n)$.

C $f \in \Theta(n)$ and $g \in \Theta(n) \Rightarrow f + g \in \Theta(n)$.

D None of these claims are true.

E All of these claims are true.

(i) Suppose $f(x) > 0$ for all x, and $f(i+1)/f(i) \leq r$, where $0 < r < 1$. For which of the following g is $\sum_{i=1}^{n} f(i) \in \Theta(g(n))$?

A $g(n) = 1$.

B $g(n) = 2^r$.

C $g(n) = \ln(r)$.

D $g(n) = r^n$.

E None of the above.

(j) Which of these sums are $O(n^2)$: (a) $\sum_{i=1}^{n}(1+i)^2$ (b) $\sum_{i=1}^{n} 2^i$ (c) $\sum_{i=1}^{n} \frac{i}{1+i^2}$ (d) $\sum_{i=1}^{n}(-1)^i i$?

A a, b

B c

C a, b, c

D a, c

E c, d

2 **Let** $d = \mathbf{gcd}(m, n)$**, where** $m, n > 0$. **Bezout's Theorem gives** $d = mx + ny$ **where** $x, y \in \mathbb{Z}$**. Prove or disprove that it is always possible to find** $a, b \in \mathbb{Z}$ **for which** $ax + by = 1$.

3 **A leaf is a vertex with degree 1. Let Δ denote the maximum degree in a tree T. Use the hand-shaking theorem to prove that T has at least Δ leaves.**

4 Prove, or disprove: $n! \in \Theta(2^n)$.

5 For $k \in \mathbb{N}$, show that $2^k + 1$ and $2^k - 1$ are relatively prime.

6 **Let $n \geq 1$ be a natural number. Prove that $2^{(1/2)^n}$ is not rational.**

SCRATCH

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