

# ASSIGNMENT 1

## 1. WARM UP (DO BEFORE RECITATION LAB)

- (1)  $p$  and  $q$  are two propositions that are either T or F. How many rows are there in the truth table of the compound proposition  $\neg(p \vee q) \wedge \neg p$ .
- (2) Give the truth table for the compound proposition above.
- (3) **Theorem:** If integer  $n \in \mathbb{N}$  is even then  $n^2$  is even". What is wrong with the following proof:
  1. Suppose  $n^2$  is even.
  2. Let  $n$  have a prime factorization  $p_1^{q_1} p_2^{q_2} \dots$ .
  3. So in the prime factorization of  $n^2$ , each prime appears an even number of times,  $n^2 = p_1^{2q_1} p_2^{2q_2} \dots$ .
  4. Since  $n^2$  is even, 2 is a prime factor of  $n^2$ , so  $n^2 = 2^{2q} \dots$ , with  $q > 0$ .
  5. So  $n = 2^q \dots$ , with  $q > 0$  and so  $n$  has 2 as a factor and is therefore even as stated in the theorem.

What does the above proof actually prove?
- (4) Is  $n^2 + n + 41$  prime for  $n = 1, 2, 3, 4, \dots, 10$ . Does that mean that  $n^2 + n + 41$  is prime for all  $n \in \mathbb{N}$ ?
- (5) Let  $F(x)$  denote 'x is a freshman' and  $M(x)$  denote 'x is a math major'. Translate into sensible english:
  - (a)  $\forall x(M(x) \rightarrow \neg F(x))$
  - (b)  $\neg \exists x(M(x) \wedge \neg F(x))$

## 2. RECITATION LAB (TA WILL WORK THESE OUT IN LAB)

- (1) Using a truth table, determine whether these two compound propositions are logically equivalent:

$$(p \rightarrow q) \rightarrow r \qquad p \rightarrow (q \rightarrow r).$$

- (2) What is the negation of "Jan is rich and happy"
- (3) [DNF] Using only  $\neg, \wedge, \vee$ , give a compound proposition which has the following truth table:

p	q	r	
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

- (4) What is the set  $\mathbb{Z} \cap \overline{\mathbb{N}} \cap \mathcal{S}$ , where the set  $\mathcal{S}$  is the set of integers which are perfect squares.
- (5) Prove that there are infinitely many primes.

## 3. PROBLEMS (HAND THESE IN)

- (1) Use a direct proof to show that the product of two odd numbers is odd.
- (2) Using only  $\neg, \wedge, \vee$ , give a compound proposition which has the following truth table:

q	r	
T	T	F
T	F	T
F	T	F
F	F	F

- (3) Compute the *number* of positive divisors of the following integers:

6, 8, 12, 18, 30,  
4, 9, 16, 25, 36

Formulate a conjecture that relates a property of the number of divisors of  $n$  to a property of  $n$ . Your conjecture should at the very least agree with your data. State your conjecture as a theorem. You do not have to prove your theorem but be **precise** in your statement of the theorem. (You may want to define some convenient notation, for example let  $\phi(n)$  be the number of positive divisors of  $n$ ).

- (4) For the Ebola spreading model, a square gets infected if at least two (non-diagonal) squares are infected. Show the final state of the grid (who is infected) when the shaded in squares are initially infected.

FIGURE NOT INCLUDED

Can you find an arrangement of 5 initial infections that can eventually infect the whole  $6 \times 6$  square. What about with 6 initial infections? Try the same game with a  $4 \times 4$  square and a  $5 \times 5$  square.

Suppose the square is  $n \times n$  where  $n \in \mathbb{N}$ . Formulate a conjecture that asserts the minimum number of initial infections required to eventually infect the whole square.

State your conjecture as a theorem. You do not have to prove your theorem but be **precise** in your statement of the theorem.

- (5) Prove or Disprove each of the following theorems:

**Theorem.** If  $x$  and  $y$  are both *irrational*, then  $x^y$  is irrational.

[Hint: We proved in class that  $\sqrt{2}$  is irrational.]

**Theorem.** If  $n$  is an integer and  $n^2$  is divisible by 3, then  $n$  is divisible by 3.