Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

**GOOD LUCK!**

You **MUST** show **CORRECT** work to get full credit.

When in doubt, **TINKER**.

<table>
<thead>
<tr>
<th>1-12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>10 each</strong></td>
<td><strong>120</strong></td>
</tr>
</tbody>
</table>
Determine the type of proof and prove: a right triangle with integer sides cannot be isoceles.
2 Prove that the product of any 5 consecutive natural numbers is divisible by $5!$. 
3. Let \( G_0 = 0 \), \( G_1 = 1 \), and \( G_n = 7G_{n-1} - 12G_{n-2} \) for \( n > 1 \). Compute \( G_5 \). Show \( G_n = 4^n - 3^n \) for \( n \geq 0 \).
Determine and prove the order-relationships between $\ln n$, $\ln(n^2 + 1)$, and $\ln(2n)$. 
5 Prove that

\[ xk \equiv yk \mod d \]

implies that

\[ x \equiv y \mod \left(\frac{d}{\gcd(k, d)}\right). \]
Adam, Barb, Charlie, and Doris each independently choose a random number uniformly distributed in \{1, 2, 3, 4, 5\}. What is the probability that some pair chooses the same number? What if there are \(k\) people and \(n\) numbers?
Let \( V \) be a set of \( n \) vertices, and let the edge set \( \mathcal{E} \) be initially empty. For each pair of vertices \( i \neq j \), add the edge \((i, j)\) to \( \mathcal{E} \) with probability \( p \). Give the pdf for the degrees of the nodes of this graph.
Voltage in the US has a mean of 120V and a standard deviation of 5V. A device’s operating voltage is 112–128. Use Chebyshev’s inequality to bound the probability that the device will not be damaged when turned on.
Give a DFA for strings whose even digits alternate between 0 and 1.
For CFG $S \rightarrow 0S|S1|0|1$, prove that no string has 10 as a substring.
Consider the language of palindromes $L = \{ \omega \mid \omega \in \{0,1\}^*, \omega = \omega^R \}$. Give well-written high-level pseudocode for a decider for this language.
Given an ultimate-debugger $D$ that takes $\langle M \rangle \# \omega$ and decides if TM $M$ halts on input $\omega$, show that every recognizer of a language $L$ can be converted into a decider for the language.
SCRATCH