QUIZ 1: 60 Minutes

Last Name: ______________________
First Name: ______________________
RIN: ____________________________
Section: _________________________

Answer ALL questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

You MUST show CORRECT work to get full credit.

When in doubt, TINKER.

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1 Circle one answer per question. 20 points for each correct answer.

(a) Identify the converse of the statement “If I have time and I am not too tired, then I will go to the gym”.

A If I will not go to the gym, then I have time and I am not too tired.
B If I will not go to the gym, then I do not have time or I am too tired.
C If I will not go to the gym, then I do not have time and I am too tired.
D If I will go to the gym, then I have time and I am not too tired.
E None of the above.

(b) Which type of proof is most appropriate to establish that the product of an irrational number and a rational number is irrational?

A Direct.
B Contraposition.
C Induction.
D Contradiction.
E None of the above.

(c) Which of the following claims are true?

(1) \( \neg (\forall x : P(x)) \equiv \exists x : \neg P(x) \)
(2) \( P(x) \lor Q(x) \lor \neg P(x) \equiv Q(x) \)
(3) \( p \rightarrow q \equiv \neg q \lor p \)

A (1).
B (1) and (2).
C (1) and (3).
D (2).
E (2) and (3).
(d) Which method of proof is most appropriate for establishing that for all positive integers \( n \geq 2 \),
\[ 1 + 2^n < 3^n \]?

A Contradiction.
B Contraposition.
C Direct.
D Induction.
E None of the above.

(e) What is the contrapositive of the statement “If I drive without first warming up the car, the engine
doesn’t start or it shakes”?

A If the engine doesn’t start or it shakes, then I drove without first warming up the car.
B If the engine starts and doesn’t shake, then I drove without first warming up the car.
C If the engine starts and doesn’t shake, then I drove after first warming up the car.
D If the engine doesn’t start or it shakes, then I drove after first warming up the car.
E None of the above.
Let $x, y, a$ be positive numbers with $x \leq y$. Prove that

\[
\frac{x + a}{y + a} \geq \frac{x}{y}.
\]
Prove that the sum of any five consecutive natural numbers is divisible by 5.