Deontic Cognitive Event Calculus
(Formal Specification)

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$\mathcal{DCEC}^*$ (deontic cognitive event calculus) is a *multi-sorted quantified modal logic*[1] that has a well-defined syntax and a proof calculus. The syntax of the language of $\mathcal{DCEC}^*$ and the rules of inference for its proof calculus are shown in Figure 1. $\mathcal{DCEC}^*$ syntax includes a system of sorts S, a signature f, a grammar for terms t, and a grammar for sentences ϕ; these are shown on the left half of the figure. The proof calculus is based on natural deduction (Jaśkowski 1934), and includes all the introduction and elimination rules for first-order logic, as well as rules for the modal operators; the rules are listed in the right half of the figure.

The formal semantics for $\mathcal{DCEC}^*$ is still under development; a semantic account of the wide array of cognitive and epistemic constructs found in the logic is no simple task — especially because of two self-imposed constraints: resisting fallback to the standard ammunition of possible-worlds semantics (which for reasons beyond the scope of the present paper we find manifestly implausible as a technique for formalizing the meaning of epistemic operators), and resisting the piggybacking of deontic operators on pre-established logics not expressly created and refined for the purpose of doing justice to moral reasoning in the human realm.[2]

Of course, we have informal interpretations for the different "parts of speech" in $\mathcal{DCEC}^*$: We denote that agent a knows ϕ at time t by $\mathbf{K}(a,t,\phi)$. The operators $\mathbf{B}$ and $\mathbf{P}$ have a similar informal interpretation for belief and perception, respectively. $\mathbf{D}(a,t,holds(f,t'))$ says that the agent a at time t desires that the fluent f holds at time t'. The formula $\mathbf{S}(a,b,t,\phi)$ captures declarative communication of ϕ from agent a to agent b at time t. Public declaration of ϕ at time t by a is denoted by $\mathbf{S}(a,t,\phi)$. Common-knowledge of ϕ in the system at time t is denoted by $\mathbf{C}(t,\phi)$. Common-knowledge of some proposition ϕ holds exactly when every agent knows ϕ, and every agent knows that every agent knows ϕ, and so on *ad infinitum*. Note the restrictions on the form of $\mathbf{I}$ and $\mathbf{O}$. $\mathbf{I}(a,t,happens(action(a^*,\alpha),t'))$ indicates that the agent a at time t intends to perform an action of type α at some time t'; the $*$ operator, written here in postfix form, ensures that this is an exact self-referential attitude, and not an attitude that happens to hold of the same agent by happenstance.

[1]Manzano (1996) covers muli-sorted first-order logic (MSL). Details as to how a reduction of intensional logic to MSL so that automated theorem proving based in MSL can be harnessed is provided in (Arkoudas & Bringsjord 2009).

[2]Such piggybacking is the main driver of (Horty 2012), in which deontic logic is understood via aligning it with default logic.

Figure 1: Deontic Cognitive Event Calculus

Syntax

$$S ::= \textsf{Object} \mid \textsf{Agent} \mid \textsf{Self} \sqsubset \textsf{Agent} \mid \textsf{ActionType} \mid \textsf{Action} \sqsubseteq \textsf{Event} \mid \textsf{Moment} \mid \textsf{Boolean} \mid \textsf{Fluent} \mid \textsf{Numeric}$$

$$f ::= \begin{array}{l} \mathit{action} : \textsf{Agent} \times \textsf{ActionType} \rightarrow \textsf{Action} \\ \mathit{initially} : \textsf{Fluent} \rightarrow \textsf{Boolean} \\ \mathit{holds} : \textsf{Fluent} \times \textsf{Moment} \rightarrow \textsf{Boolean} \\ \mathit{happens} : \textsf{Event} \times \textsf{Moment} \rightarrow \textsf{Boolean} \\ \mathit{clipped} : \textsf{Moment} \times \textsf{Fluent} \times \textsf{Moment} \rightarrow \mathit{Boolean} \\ \mathit{initiates} : \textsf{Event} \times \textsf{Fluent} \times \textsf{Moment} \rightarrow \textsf{Boolean} \\ \mathit{terminates} : \textsf{Event} \times \textsf{Fluent} \times \textsf{Moment} \rightarrow \textsf{Boolean} \\ \mathit{prior} : \textsf{Moment} \times \textsf{Moment} \rightarrow \textsf{Boolean} \\ \mathit{interval} : \textsf{Moment} \times \textsf{Boolean} \\ * : \textsf{Agent} \rightarrow \textsf{Self} \\ \mathit{payoff} : \textsf{Agent} \times \textsf{ActionType} \times \textsf{Moment} \rightarrow \textsf{Numeric} \end{array}$$

$$t ::= x : S \mid c : S \mid f(t_1, \ldots, t_n)$$

$$\phi ::= \begin{array}{l} p : \textsf{Boolean} \mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid \phi \leftrightarrow \psi \mid \forall x : S.\ \phi \mid \exists x : S.\ \phi \\ \mathbf{P}(a,t,\phi) \mid \mathbf{K}(a,t,\phi) \mid \mathbf{C}(t,\phi) \mid \mathbf{S}(a,b,t,\phi) \mid \mathbf{S}(a,t,\phi) \\ \mathbf{B}(a,t,\phi) \mid \mathbf{D}(a,t,\mathit{holds}(f,t')) \mid \mathbf{I}(a,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t')) \\ \mathbf{O}(a,t,\phi,\mathit{happens}(\mathit{action}(a^*,\alpha),t')) \end{array}$$

Rules of Inference

$$\frac{}{\mathbf{C}(t,\mathbf{P}(a,t,\phi) \rightarrow \mathbf{K}(a,t,\phi))}\ [R_1] \qquad \frac{}{\mathbf{C}(t,\mathbf{K}(a,t,\phi) \rightarrow \mathbf{B}(a,t,\phi))}\ [R_2]$$

$$\frac{\mathbf{C}(t,\phi)\ t \leq t_1 \ldots t \leq t_n}{\mathbf{K}(a_1,t_1,\ldots \mathbf{K}(a_n,t_n,\phi)\ldots)}\ [R_3] \qquad \frac{\mathbf{K}(a,t,\phi)}{\phi}\ [R_4]$$

$$\frac{t_1 \leq t_3, t_2 \leq t_3}{\mathbf{C}(t,\mathbf{K}(a,t_1,\phi_1 \rightarrow \phi_2) \rightarrow (\mathbf{K}(a,t_2,\phi_1) \rightarrow \mathbf{K}(a,t_3,\phi_2)))}\ [R_5]$$

$$\frac{t_1 \leq t_3, t_2 \leq t_3}{\mathbf{C}(t,\mathbf{B}(a,t_1,\phi_1 \rightarrow \phi_2) \rightarrow (\mathbf{B}(a,t_2,\phi_1) \rightarrow \mathbf{B}(a,t_3,\phi_2)))}\ [R_6]$$

$$\frac{t_1 \leq t_3, t_2 \leq t_3}{\mathbf{C}(t,\mathbf{C}(t_1,\phi_1 \rightarrow \phi_2) \rightarrow (\mathbf{C}(t_2,\phi_1) \rightarrow \mathbf{C}(t_3,\phi_2)))}\ [R_7]$$

$$\frac{}{\mathbf{C}(t,\forall x.\ \phi \rightarrow \phi[x \mapsto t])}\ [R_8] \qquad \frac{}{\mathbf{C}(t,\phi_1 \leftrightarrow \phi_2 \rightarrow \neg\phi_2 \rightarrow \neg\phi_1)}\ [R_9]$$

$$\frac{}{\mathbf{C}(t,[\phi_1 \wedge \ldots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \ldots \rightarrow \phi_n \rightarrow \psi])}\ [R_{10}]$$

$$\frac{\mathbf{B}(a,t,\phi)\ \ \mathbf{B}(a,t,\phi \rightarrow \psi)}{\mathbf{B}(a,t,\psi)}\ [R_{11a}] \qquad \frac{\mathbf{B}(a,t,\phi)\ \ \mathbf{B}(a,t,\psi)}{\mathbf{B}(a,t,\psi \wedge \phi)}\ [R_{11b}]$$

$$\frac{\mathbf{S}(s,h,t,\phi)}{\mathbf{B}(h,t,\mathbf{B}(s,t,\phi))}\ [R_{12}]$$

$$\frac{\mathbf{I}(a,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t'))}{\mathbf{P}(a,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t))}\ [R_{13}]$$

$$\frac{\begin{array}{c}\mathbf{B}(a,t,\phi)\ \ \mathbf{B}(a,t,\mathbf{O}(a^*,t,\phi,\mathit{happens}(\mathit{action}(a^*,\alpha),t'))) \\ \mathbf{O}(a,t,\phi,\mathit{happens}(\mathit{action}(a^*,\alpha),t'))\end{array}}{\mathbf{K}(a,t,\mathbf{I}(a^*,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t')))}\ [R_{14}]$$

$$\frac{\phi \leftrightarrow \psi}{\mathbf{O}(a,t,\phi,\gamma) \leftrightarrow \mathbf{O}(a,t,\psi,\gamma)}\ [R_{15}]$$

This representation closely follows that of Casta˜neda, and a more elaborate account of self-reference in $\mathcal{DCEC}^*$can be found in (Bringsjord & Govindarajulu 2013). The latest addition to the calculus is the *ought-to-be* dyadic deontic operator $\mathbf{O}$ presented in (Goble 2003, McNamara 2010), intended to help dodge Chisholm's paradox, which plagues Standard Deontic Logic (SDL).[3] $\mathbf{O}(a,t,\phi,\mathit{happens}(\mathit{action}(a^*,\alpha),t'))$ is to be read as: *"if it is the case that a at time t believes* ϕ *then that* α *is obligatory for a and this is known by a*." The $\textsf{Moment}$ sort is used for representing time-points. We assume that time-points are isomorphic with $\mathbb{N}$; and function symbols (or functors) $+, -$; and relation symbols $>, <, \geq, \leq$ are available, under standard interpretations.

$\mathcal{DCEC}^*$has a classical monotonic view of the knowledge of the world possessed by an agent and views knowledge to be true and unchanging. This means that if an agent knows ϕ at some time t, then the agent will continue to know ϕ for all time. Beliefs possessed by an agent can change as time passes and events happen and fluents change, but knowledge remains constant or increases. This view of knowledge underpins all inference rules that have a knowledge component. For example, $[R_3]$ states that if some information ϕ is common knowledge at a certain time, then we can derive that an agent knows that at a certain time that another agent knows at another time, and so on, until finally, that an agent knows ϕ at a time, with all these time indexicals occurring later than the moment at which the common knowledge holds. Rule R_{15} for $\mathbf{O}$ is based on the only rule for the ought-to-be operator in (Goble 2003, McNamara 2010), which can be easily

[3]An excellent overview of SDL and Chisholm's Paradox available in the Stanford Encyclopedia of Philosophy; see http://plato.stanford.edu/entries/logic-deontic.

and plausibly interpreted as also holding for the ought-to-do case. Rule R_{14} connects the $\mathbf{O}$ operator with the knowledge and belief operators. The rule is to be informally read as follows: *"If it is the case that an agent believes that the agent ought to* α *when* ϕ *holds at any time, and it is the case that the agent ought to* α *when* ϕ*, and the agent believes that* ϕ *holds at a given time, then the agent knows that the agent intends to perform action* α*."*.[4]

The rule for the communication operator is from the analysis in (Wooldridge 2009, Chapter 7); the rules for the rest of the modal operators come from (Arkoudas & Bringsjord 2008*a*) and (Bringsjord & Govindarajulu 2013). Rules R_{11a} and R_{11b} enable an agent which believes in $\{\phi_2,\ldots,\phi_n\}$ to also believe ψ if $\{\phi_1,\ldots,\phi_n\}\vdash\psi$. Some readers may be uncomfortable with the duo of R_{11a} and R_{11b}, but we have included them as this is not only realistic but also necessary when agents represent nations.

$\mathcal{DCEC}^*$ includes the signature of the classic Event Calculus ($\mathcal{EC}$) (Mueller 2006),[5] and the axioms of EC are assumed to be common knowledge in the system. EC is a first-order calculus that lets one reason about events that occur in time and their effects on fluents. $\mathcal{DCEC}^*$ includes, in addition to symbols for basic arithmetic functions and relations $S_{ar}=\{0,1,+,.,<,>\}$, a relevant theory of arithmetic Φ_{arith}. The details vary from application to application; the power of this theory can be altered. This allows us to model agents that can access numerical propositions and calculations; for example, an agent using simple utilitarian calculation to ground its decisions. The agents are also assumed to have some basic knowledge of causality going beyond the EC; this is expressed as common knowledge. These axioms are not pertinent to the present study; they can be found in (Arkoudas & Bringsjord 2008*b*).

$\mathcal{DCEC}^*$ has a set of distinguished constant symbols corresponding to when and by whom the reasoning is carried out: now is a symbol indicating the current time, and I is a symbol indicating the agent carrying out the reasoning. These features have not yet been fully developed, but the need for these features is explained in (Bringsjord & Govindarajulu 2013).

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[4] The complexity of R14 is perhaps well beyond what is needed for present purposes (the mere introduction of our framework and general methodology). For example, it would be simpler, and probably defensible in the context of the present prolegomenon, to simply go with $\dfrac{\mathbf{B}(a,t,\phi)\quad \mathbf{O}(a,t,\phi,\gamma)}{\mathbf{K}(a,t,\mathbf{I}(a^*,t,\gamma))}$, where $\gamma\equiv happens(action(a^*,\alpha),t')$.

[5] Let the infix $t_1<t2$ stand for $prior(t_1,t_2)$; then the axioms of the EC are:

$$\forall f:\mathsf{Fluent},\ t:\mathsf{Moment}.\ initially(f)\land\neg clipped(0,f,t)\Rightarrow holds(f,t)$$

$$\forall e:\mathsf{Event},\ f:\mathsf{Fluent},\ t_1,t_2:\mathsf{Moment}.\ happens(e,t_1)\land initiates(e,f,t_1)\land t_1<t_2\land\neg clipped(t_1,f,t_2)\Rightarrow holds(f,t_2)$$

$$\forall t_1,t_2:\mathsf{Moment}, f:\mathsf{Fluent}.\ clipped(t_1,f,t_2)\Leftrightarrow(\exists e:\mathsf{Event},\ t:\mathsf{Moment}.\ happens(e,t)\land t_1<t<t_2\land terminates(e,f,t))$$

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