Making Precise Statements
Reading

  - Chapter 3
Today

• Making a precise statement: the proposition
• Complicated precise statements: the compound proposition
  – Truth tables
• Claims about many things
  – Predicates
  – Quantifiers
  – Proofs with quantifiers
Statements can be ambiguous

• Precise statements
  \[ 2 + 2 = 4 \] (True)
  \[ 2 + 2 = 5 \] (False)

• Not-so-precise statements
  – You can have ice cream or cake
    • Can I have both?
    • Exclusive or Inclusive Or?
  – If pigs can fly, then you get an A
    • Pigs can’t fly, so do you still get an A?
    • False \(\rightarrow\) Anything
  – There is a room for every student
    • Do all students share the same room?
    • Does each student get an individual room?
Why is ambiguity bad?

• We want to **prove** things!
• Need to know when and if computers implement correct algorithms!
• Beware of ambiguous statements
  – Natural language is ambiguous by design
  – That’s why we have math
Propositions are True (T) or False (F)

- Propositions are represented using lowercase letters $p, q, r, s, ...$
- Piglet can fly
  - False
- You got an A
  - Hmm.. T?
- $4^2$ is even
  - True

- There are actually many types of logics out there
  - E.g., fuzzy logic includes probabilities
  - There are logics that also include a third value, Maybe/Don’t know
  - We are only going to focus on classical logic
    - If something is not T, then it must be F
Compound Statements

• Piglet can fly OR $4^2$ is even
  – True

• Piglet can fly $\rightarrow$ You got an A
  – True
  – False $\rightarrow$ anything

• Piglet cannot fly $\rightarrow$ You got an A
  – ?
  – Depends on the value of “You got an A”
Notation

- Conjunction
  \[ p \land q \]
  \[ p \text{ AND } q \]
- Disjunction
  \[ p \lor q \]
  \[ p \text{ OR } q \]
- Negation
  \[ \neg p \]
  \[ \text{NOT } p \]
- Implication
  \[ p \rightarrow q \]
  \[ p \text{ IMPLIES } q \]
Negation

• The negation $\neg p$ is F when $p$ is T
• The negation $\neg p$ is T when $p$ is F
• Piglet can fly is F
• $\neg$(Piglet can fly) is T
• Notice how English quickly becomes redundant/ambiguous
  – Piglet cannot fly
  – It is not the case that Piglet can fly
Conjunction

- Both $p$ and $q$ must be $T$ for $p \land q$ to be $T$
  - Otherwise $p \land q$ is $F$
- Piglet can fly AND You got an A
  - $F$(alse)
  - (Piglet can fly) $\land$ (You got an A) = $F$
- Piglet cannot fly AND You got an A
  - $?$
  - Depends on the value of (You got an A)
Disjunction

• Both $p$ and $q$ must be F for $p \lor q$ to be F
  – Otherwise it is T

• (Piglet cannot fly) $\lor$ (You got an A)
  – Depends on the value of (You got an A)

• $\neg$(Piglet cannot fly) $\lor$ (You got an A) = T
  – Why?
  – Because $\neg$(Piglet cannot fly) = T

• (You can have cake) $\lor$ (You can have ice-cream)
  – Can you have both?
  – Yes, this is Inclusive OR
  – Exclusive OR is true when exactly one is true
Truth Table

- Essentially a function that maps the value of $p$ and $q$ to the statement we’re trying to make
- Defines the meaning of these operators

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- Can also use this in the case of any logic formula
Implication

• Piglet can fly → You got an A
  – IF Piglet can fly THEN You got an A
• IF $n^2$ is even, THEN $n$ is even
  – Is every even square the square of an even number?
• IF (it rained last night) THEN (the grass is wet)
  $p = \text{(it rained last night)}$
  $q = \text{(the grass is wet)}$
  – In logic notation: $p \rightarrow q$
  – What does it *mean* for this common-sense implication to be true?
    • We have built a model of the world
    • Whenever we observe $p$, we can make conclusions about $q$
    • If we don’t observe $p$, our model tells us nothing about $q$
    • If only observe $q$, can’t conclude anything about $p$
  – What can you conclude? Did it rain last night? Is the grass wet?
Implication, cont’d

• IF (it rained last night) THEN (the grass is wet)
  – What does it mean for this common-sense implication to be true?
  – What can you conclude? Did it rain last night? Is the grass wet?

• Suppose you look at the weather report for last night, and it indeed rained

• Is the grass wet?
  – YES

• For a true implication $p \rightarrow q$, you can conclude $q = T$ when $p = T$
Implication, cont’d

• IF (it rained last night) THEN (the grass is wet)
  – What does it mean for this common-sense implication to be true?
  – What can you conclude? Did it rain last night? Is the grass wet?

• Suppose you see wet grass in the morning
  – Did it rain?
  – Can’t tell

• For a true implication $p \rightarrow q$, when $q = T$ you cannot conclude $p = T$
Implication, cont’d

• IF (it rained last night) THEN (the grass is wet)
  – What does it mean for this common-sense implication to be true?
  – What can you conclude? Did it rain last night? Is the grass wet?

• Suppose you see dry grass in the morning
  – Did it rain?
  – No
    • Our model of the world assumes the grass MUST BE wet if it rained

• For a true implication $p \rightarrow q$, when $q$ is F, you can conclude $p$ is F
Implication, cont’d

• IF (it rained last night) THEN (the grass is wet)
  – What does it mean for this common-sense implication to be true?
  – What can you conclude? Did it rain last night? Is the grass wet?
• Suppose you see no rain in the weather report
  – Is the grass wet?
  – Can’t tell
• For a true implication $p \rightarrow q$, when $p$ is F, you cannot conclude $q$ is F
Implication: inferences when new information comes

For a true implication $p \rightarrow q$:

- When $p$ is $T$, you can conclude that $q$ is $T$.
- When $q$ is $T$, you **cannot** conclude $p$ is $T$.
- When $p$ is $F$, you **cannot** conclude $q$ is $F$.
- When $q$ is $F$, you can conclude $p$ is $F$. 
Falsifying IF (it rained last night) THEN (the grass is wet)

• You are a scientist collecting data to verify that the implication is valid (true)

• One night it rained. That morning the grass was dry.
  – New information

• What do you think about the implication now?

• This is a falsifying scenario
  – IF (it rains) THEN (the grass is wet)
  – False

• Our model of the world was wrong

• \( p \rightarrow q \) is only F when \( p = T \) and \( q = F \)
  – In all other cases, \( p \rightarrow q = T \)
Implication is EXTREMELY important

• All these are $p \rightarrow q$ ($p =$ “it rained last night” and $q =$ “the grass is wet”):  
  – If it rained last night then the grass is wet (IF $p$ THEN $q$)  
  – It rained last night implies the grass is wet (p IMPLIES q)  
  – It rained last night only if the grass is wet (p ONLY IF q)  
  – The grass is wet if it rained last night (q IF p)  
  – The grass is wet whenever it rains (q WHenever p)  

• Notice that there are multiple English descriptions the same logical statement
Implication Truth Table

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Example

- IF (you are hungry OR you are thirsty) THEN you visit the cafeteria
  - \((p \lor q) \rightarrow r\)
    - where \(p = \text{you are hungry}, q = \text{you are thirsty}, r = \text{you visit the cafeteria}\)
- You are thirsty: \(q\) is T.
  - There are two rows where \(q\) is T and \((p \lor q) \rightarrow r\) is T
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• Order is very important!
  – In particular, $p \rightarrow q$ and $q \rightarrow p$ do not mean the same thing!
• IF I’m dead, THEN my eyes are closed vs. IF my eyes are closed, THEN I’m dead
Proving an Implication: Reasoning without Facts

• IF \( n^2 \) is even \THEN \( n \) is even

\[
\begin{array}{|c|c|c|}
\hline
p & q & p \rightarrow q \\
\hline
F & F & T \\
F & T & T \\
T & F & F \\
T & T & T \\
\hline
\end{array}
\]

• What is \( n \)? How to prove?
  – We must show that the highlighted row \textit{cannot} occur.
  – i.e., \( n \) is odd cannot be the case

• In this row, \( q \) is \( F \): \( n = 2k + 1 \)

• \( n^2 = (2k + 1)^2 = 2(2k^2 + 2k) + 1 \)

• \( p \) cannot be \( T \). This row cannot happen: \( p \rightarrow q \) is always \( T \)
Quantifiers

- Every person has a soulmate
- John has some gray hair
- Everyone has some gray hair
- Any map can be colored with 4 colors with adjacent countries having different colors
- Every even integer \( n > 2 \) is the sum of 2 primes \((\text{Goldbach conjecture, 1742})\)
  - Still not proven, but holds for numbers up to at least \( 4 \times 10^{18} \)
- Someone broke this faucet
- There exists a creature with blue eyes and blonde hair
- All cars have four wheels
Quantifiers, etc.

• These statements are more complex because of quantifiers:
  – EVERY; A; SOME; ANY; ALL; THERE EXISTS

• Compare:
  – My Tesla has four wheels
  – ALL cars have four wheels
Predicates are like functions

- ALL cars have four wheels
- Define predicate $P(c)$ and its domain
  - $C = \{c \mid c \text{ is a car}\}$
    - set of cars
  - $P(c) = \text{“car c has four wheels”}$
- “for all $c$ in $C$, the statement $P(c)$ is true.”
  - $\forall c \in C: P(c)$
    - ($\forall$ means “for all”)

\[\]
Predicates are like functions, cont’d

• Predicates

\[ P(c) = \text{“car } c \text{ has four wheels”} \]

– Input: parameter \( c \in C \)
– Output: statement \( P(c) \)
– Example: \( P(Jen's \ Car) = \text{“car Jen's Car has four wheels”} \)

\[ \forall c \in C: P(c) \]

– Meaning: for all \( c \in C \), the statement \( P(c) \) is T

• Functions:

\[ f(x) = x^2 \]

– Input: parameter \( x \in \mathbb{R} \)
– Output: value \( f(x) \)
– Example: \( f(5) = 25 \)

\[ \forall x \in \mathbb{R}, f(x) \geq 0 \]

– Meaning: for all \( x \in \mathbb{R} \), \( f(x) \) is \( \geq 0 \)
Example

• There EXISTS a creature with blue eyes and blonde hair
• Define predicate $Q(a)$ and its domain
  $$A = \{a | a \text{ is a creature}\}$$
  – set of creatures
  $$Q(a) = \text{“}a \text{ has blue eyes and blonde hair”}$$
• “there exists $a$ in $A$ for which the statement $Q(a)$ is true.”
  $$\exists a \in A: Q(a)$$
  $\exists$ means “there exists”
• $G(a) = \text{“}a \text{ has blue eyes”}$
• $H(a) = \text{“}a \text{ has blonde hair”}$
• $\exists a \in A: (G(a) \land H(a))$
Example, cont’d

• \( G(a) = “a \text{ has blue eyes}” \)
• \( H(a) = “a \text{ has blonde hair}” \)
• \( \exists a \in A: (G(a) \land H(a)) \)
  – compound predicate
• When the domain is understood, we don’t need to keep repeating it
  – We write

\[ \exists a: Q(a) \]

– or

\[ \exists a: (G(a) \land H(a)) \]
Negative Quantifiers

• IT IS NOT THE CASE THAT (There EXISTS a creature with blue eyes and blonde hair)
  
• Same as: “All creatures don’t have blue eyes and blonde hair”
  \[ \neg (\exists a \in A: Q(a)) \equiv \forall a \in A: \neg Q(a) \]
  (\equiv \text{ means they are equivalent/same})

• IT IS NOT THE CASE THAT (All cars have four wheels)
  
• Same as: “There is a car which does not have four wheels”
  \[ \neg (\forall c \in C: P(c)) \equiv \exists c \in C: \neg P(c) \]

• When you take the negation inside the quantifier and negate the predicate, you must switch quantifiers:
  \[ \exists \rightarrow \forall \]
  \[ \forall \rightarrow \exists \]
There is a soulmate for EVERY person

- Define domains and a predicate
  \[ A = \{ a | a \text{ is a person} \} \]
- \( P(a, b) = \text{“Person } a \text{ has as a soul mate person } b \text{”} \)
- There is some special person \( b \) who is a soul mate to every person \( a \)
  \[ \exists b: (\forall a: P(a, b)) \]
- For every person \( a \), they have their own personal soul mate \( b \)
  \[ \forall a: (\exists b: P(a, b)) \]
- When quantifiers are mixed, the order in which they appear is important for the meaning
  - Order generally cannot be switched
Proofs with quantifiers

• **Claim 1.** \( \forall n > 2: \) if \( n \) is even, then \( n \) is a sum of two primes. *(Goldbach, 1742)*

• **Claim 2.** \( \exists (a, b, c) \in \mathbb{N}^3: a^2 + b^2 = c^2 \)
  
  – where \( (a, b, c) \in \mathbb{N}^3 \) means triples of natural numbers

• **Claim 3.** \( \neg \exists (a, b, c) \in \mathbb{N}^3: a^3 + b^3 = c^3 \)

• **Claim 4.** \( \forall (a, b, c) \in \mathbb{N}^3: a^3 + b^3 = c^3 \)

• Think about what it would take to prove these claims