Quiz 2

60 Minutes

First Name: Solutions

Last Name: ______________________

RIN: ______________________

NO COLLABORATION or electronic devices.
Any violations will result in an F.
No questions allowed during the test unless you think there is a mistake.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You MUST show CORRECT work to get credit.
Correct answers with no explanation will get a 0.

Final Score: _____ / 200
1. Suppose a goody-bag contains 3 candies. Candies come in three colors: red, green and blue. How many types of goody-bags are there in total?

A \( \binom{3}{2} \)  
B 3!  
C \( \binom{3}{2} \)  
D 5!  
E None of the above.

Three 0's, 2 delimiters
Five bits total

2. Suppose a goody-bag contains 3 candies. Candies come in three colors: red, green and blue. If all goody-bags are equally likely, how many goody-bags would I need to buy in order to guarantee I have a candy of each color?

A \( \binom{3}{2} \)  
B 3!  
C \( \binom{3}{2} \)  
D 5!  
E None of the above.

For any finite number, there's a non-zero probability to only have 2 colors.

3. Suppose FOCS has 10 students and every student tries to shake hands with as many other students as possible. How many handshakes need to occur in total to guarantee a repeat?

A 10  
B 20  
C 35  
D 46  
E 91

\[ \text{# handshakes} = \binom{10}{2} = \frac{10!}{2!8!} = \frac{10 \times 9}{2} = 45 \]

4. Let \( X \) be a random variable and let the set of all outcomes be \( \Omega \). What is \( \sum_{x \in \Omega} P[X = x] \)?

A 0.5  
B 1  
C 1.5  
D 2  
E None of the above.

Sum of all outcomes is 1.

5. Suppose \( X(\Omega) = \{1, 2, 3\} \) and suppose \( P[X = 1 \lor X = 2] = 0.5 \). What is \( P[X = 3] \)?

A 0.2  
B 0.3  
C 0.4  
D 0.5  
E 1

\[ P[X = 3] = 1 - P[X \neq 3] \]
\[ = 1 - P[X = 1 \lor X = 2] \]
\[ = 0.5 \]
6. Suppose I toss three coins independently. What do we know?

A. At least two coins must match.
B. The probability that all coins match is $1/8$.
C. The probability of at least one H is 1.
D. The probability of at least one T is 1.
E. None of the above.

7. Suppose $X_1$ and $X_2$ are independent and uniform on $\{1, 2, 3, 4, 5\}$. What is $P[X_1 + X_2 \leq 3]$?

A. $\frac{1}{25}$
B. $\frac{2}{25}$
C. $\frac{3}{25}$
D. $\frac{4}{25}$
E. $\frac{5}{25}$

8. Suppose $X_1$ and $X_2$ are independent and uniform on $\{1, 2, 3, 4, 5\}$. What is $E[X_1 + X_2]$?

A. $3$
B. $4$
C. $5$
D. $6$
E. $7$

9. Suppose $X_1$ is uniform on $\{1, 2, 3, 4, 5\}$. If $X_1 \geq 4$, then $X_2$ is uniform on $\{4, 5\}$; otherwise $X_2 = 5$. What is $E[X_1 + X_2]$?

A. $\frac{30}{5}$
B. $\frac{33}{5}$
C. $\frac{36}{5}$
D. $\frac{39}{5}$
E. $\frac{42}{5}$

10. Suppose Submitty had a bug and randomly shuffled Quiz 2 grades. Assuming there are 200 students and all grades are different, what is the expected number of students who get their correct grade in Submitty?

A. $1$
B. $10$
C. $20$
D. $50$
E. None of the above.

$X_i = 1$ if student $i$ got their correct grade.

$X = X_1 + \cdots + X_{200}$ (all students get their correct grades).

$E[X_i] = \frac{1}{200}$ (grades randomly distributed). So $E[X] = \frac{200}{200} = 1$.
11. Suppose the correct answer is not E on any of the 20 questions and you guess randomly among A-D. How many of the 20 questions do you expect to get right?

A 1  \[ P[\text{success}] = \frac{1}{4}. \]
B 2  \[ E[20 \text{ trials}] = \frac{20}{4} = 5 \]
C 3  \[ \text{depends on the correct answer} \]
D 4  \[ \text{distribution.} \]
E 5  \[ \text{Correct answer could be always B} \]

12. Suppose you answer A on all 20 questions. How many questions do you expect to get right?

A 2  \[ \text{depends on the correct answer} \]
B 3  \[ \text{distribution.} \]
C 4  \[ \text{Correct answer could be always B} \]
D 5  \[ \text{It cannot be determined from the given information.} \]

13. Suppose it is sunny 1/10 of days in Troy. How much do you expect to wait until a sunny day?

A 5 days  \[ P[\text{success}] = \frac{1}{10} = p \]
B 10 days  \[ E[\text{waiting time}] = \frac{1}{p} = 10 \]
C 15 days
D 20 days
E None of the above.

14. Suppose it is sunny 1/10 of days in Troy. Suppose it is always sunny in Philadelphia, except for the days when it is sunny in Troy. How many days is a Philadelphian expected to wait until a sunny day?

A 10/9  \[ p[\text{success}] = \frac{9}{10} \]
B 9/10
C 2
D 3
E None of the above.

15. You are in Troy now. If the weather is not sunny, you travel to Philadelphia tomorrow; if it’s not sunny in Philadelphia tomorrow, you go back to Troy the day after (and will go back and forth on non-sunny days). How many days do you expect to wait until a sunny day (assuming same probabilities of sunny days as in Question 14)?

A 91/91  \[ B 100/91 \]
C 190/91  \[ D 290/91 \]
E 182/91

\[ P_T = \frac{1}{10} + (1 + P_T) \cdot \frac{9}{10} \]
\[ P_T = \frac{9}{10} + (1 + P_T) \cdot \frac{1}{10} \]
\[ P_T = 1 + \frac{9}{10} + \frac{9}{100} P_T \rightarrow P_T = \frac{190}{91} \]
16. Suppose a covid test is correct 90% of the time and 10% of all people have covid. What is the probability that you have covid if you tested positive?

A 1/10
B 9/10
C 1/3
D 1/2
E None of the above.

\[
P[C|Y] = \frac{P[C \cap Y]}{P[Y]} = \frac{0.9 \times 0.1}{0.9 \times 0.1 + 0.1 \times 0.9} = \frac{1}{2}
\]

17. Suppose a covid test is correct 90% of the time and 10% of all people have covid. What is the probability that you have covid if you tested positive two times independently?

A 50/100
B 50/90
C 81/100
D 81/90
E None of the above.

\[
P[C|2Y] = \frac{P[C \cap 2Y]}{P[2Y]} = \frac{\frac{81}{1000}}{\frac{81}{100}} = \frac{81}{90}
\]

18. Suppose the correct answer is uniform on \{A, B, C, D, E\}. What is the probability that at least 2 of the 20 questions have the same letter for the correct answer?

A 0
B \(\binom{20}{2}\)
C 0.5
D 1
E None of the above.

19. If each question had 26 choices, and the correct answer is uniform on \{A, \ldots, Z\}, what is the probability that at least 2 of the 20 questions have the same letter for the correct answer?

A \((\frac{25}{26})^{19} \times (\frac{24}{26})^{18} \times \cdots \times (\frac{9}{26})\)
B \((\frac{25}{26})^{19} \times (\frac{24}{26})^{18} \times \cdots \times (\frac{9}{26})\)
C \((\frac{25}{26})^{19}\)
D \(1 - (\frac{25}{26})^{19}\)
E None of the above.

10th birthday paradox problem 10th birthday problem

Correct probability 10th birthday problem 10th birthday problem 10th birthday problem

\[
1 - \left(\frac{25}{26}\right)^{19} \times \left(\frac{24}{26}\right)^{18} \times \cdots \times \left(\frac{9}{26}\right)
\]

20. \(X \sim B(p_1)\) and \(Y \sim B(p_2)\) are independent Bernoulli random variables. What is \(E[XY]\)?

A 1/4
B \(p_1 p_2\)
C \(p_1 / p_2\)
D \(p_1 + p_2\)
E None of the above.

\[E[XY] = E[X] \cdot E[Y] = p_1 p_2\]
Scratch