QUIZ 2: 60 Minutes

Last Name: Solutions
First Name: 
RIN: 
Section: 

Answer ALL questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

You MUST show CORRECT work to get full credit.

When in doubt, TINKER.
1. How many subsets of \{a, b, c, d, e\} contain the letter a?

- A 8
- B 16
- C 32
- D 64
- E None of the above.

2. How many subsets of \{a, b, c, d, e\} have exactly 3 letters?

Choose 3 from 5

\[
\binom{5}{3} = \frac{5!}{3! \cdot 2!} = \frac{5 \cdot 4 \cdot 3}{2} = 10
\]

- A 8
- B 16
- C 32
- D 64
- E None of the above.

3. All women are either young or gray or both young and gray. In a set of 50 women, 40 are young and 40 are gray. How many are both young and gray?

- A 10
- B 20
- C 30
- D 40
- E None of the above.

\[
|\text{Gray} \cup \text{Young}| = 50
\]

\[
|\text{Gray}| = 40
\]

\[
|\text{Young}| = 40
\]

\[
|\text{Gray} \cap \text{Young}| = |\text{Gray}| + |\text{Young}| - |\text{Gray} \cup \text{Young}|
\]

\[
= 40 + 40 - 50 = 30.
\]

4. You flip a 2-sided coin and roll a 6-sided die. How many outcomes are in the probability space?

- A 6
- B 8
- C 10
- D 12
- E None of the above.

\[
2 \text{ for coin} \quad 6 \text{ for die}
\]

\[
2 \times 6 = 12 \quad \text{Product Rule}
\]

5. An experiment has 3 possible outcomes \(A, B, C\). \(P(A) = 1/3\), \(P(B) = 1/6\). What is \(P(C)\)?

- A 0
- B 1/4.
- C 1/2.
- D 1.
- E None of the above.

\[
P(A) + P(B) + P(C) = 1
\]

\[
\Rightarrow P(C) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}.
\]
6. Make 3 flips of a biased coin, with probability of heads 2/3. What are the chances of more heads than tails?

\[
\begin{align*}
\text{A} & \quad 1/2. \\
\text{B} & \quad 8/9. \\
\text{C} & \quad 16/27. \\
\text{D} & \quad 20/27. \\
\text{E} & \quad \text{None of the above}
\end{align*}
\]

\[
\begin{align*}
\text{Prob} = & \quad \frac{2}{27} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{27} \\
& \quad \frac{3}{27} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}
\end{align*}
\]

\[
\text{Prob} = \frac{12}{27} + \frac{8}{27} = \frac{20}{27}
\]

7. In Problem 6, what are the chances of more heads than tails if all three flips match?

\[
\begin{align*}
\text{A} & \quad 1/2. \\
\text{B} & \quad 8/9. \\
\text{C} & \quad 16/27. \\
\text{D} & \quad 20/27. \\
\text{E} & \quad \text{None of the above}
\end{align*}
\]

\[
\begin{align*}
\text{Match} & = \text{TTT} \text{ or } \text{HHH} \\
\text{P} [\text{Match}] & = \frac{1}{27} + \frac{8}{27} = \frac{9}{27}
\end{align*}
\]

\[
\begin{align*}
\text{P} [\text{More H / Match}] & = \frac{\text{P} [\text{More H} \land \text{Match}]}{\text{P} [\text{Match}]} = \frac{\text{P} [\text{HHH}]}{\text{P} [\text{Match}]} = \frac{\frac{8}{27}}{\frac{9}{27}} = \frac{8}{9}
\end{align*}
\]

8. 1-in-20 men are color blind and 1-in-400 women are color blind. There are an equal number of men and women. What are the chances a random person is color blind?

\[
\begin{align*}
\text{A} & \quad 1/20. \\
\text{B} & \quad 20/800. \\
\text{C} & \quad 21/800. \\
\text{D} & \quad 22/800. \\
\text{E} & \quad \text{None of the above}
\end{align*}
\]

\[
\begin{align*}
\text{P} [\text{CB}] & = \frac{\text{P} [\text{CB} \mid \text{Man}] \cdot \text{P} [\text{Man}]}{\text{P} [\text{Man}]} + \frac{\text{P} [\text{CB} \mid \text{Woman}] \cdot \text{P} [\text{Woman}]}{\text{P} [\text{Woman}]} \\
& = \frac{1}{20} \cdot \frac{1}{2} + \frac{1}{400} \cdot \frac{1}{2} \\
& = \frac{1}{40} + \frac{1}{800} = \frac{21}{800}
\end{align*}
\]

9. In Problem 8, what are the chances a random color blind person is a man?

\[
\begin{align*}
\text{A} & \quad 1/20. \\
\text{B} & \quad 20/21. \\
\text{C} & \quad 20/800. \\
\text{D} & \quad 21/800. \\
\text{E} & \quad \text{None of the above}
\end{align*}
\]

\[
\begin{align*}
\text{P} [\text{Man} \mid \text{CB}] & = \frac{\text{P} [\text{Man} \land \text{CB}]}{\text{P} [\text{CB}]} = \frac{\frac{1}{20} \cdot \frac{1}{2}}{\frac{21}{800}} = \frac{20}{21}
\end{align*}
\]

10. 1-in-1000 drivers are drunk. A breathalyzer test is correct at saying if you are drunk or not 99.9% of the time. The breathalyzer says you are drunk. What are the chances you are drunk?

\[
\begin{align*}
\text{A} & \quad 1/2. \\
\text{B} & \quad 99/100. \\
\text{C} & \quad 999/1000. \\
\text{D} & \quad 999/1000. \\
\text{E} & \quad \text{None of the above}
\end{align*}
\]

\[
\begin{align*}
\text{P} [\text{Yes}] & = \frac{999}{1000} + \frac{999}{1000} \\
\text{P} [\text{Drunk} \mid \text{Yes}] & = \frac{\text{P} [\text{Drunk} \land \text{Yes}]}{\text{P} [\text{Yes}]} = \frac{999}{1000} \\
\text{P} [\text{Yes}] & = \frac{1}{2}
\end{align*}
\]
11. A box has 9 fair coins and 1 two-headed coin. Pick a random coin and flip. What are the chances of H?

A 9/20.
B 10/20.
C 11/20.
D 12/20.
E None of the above

\[
\text{PL(H)} = \text{PL(H|fair)} \cdot \text{PL(fair)} + \text{PL(H|biased)} \cdot \text{PL(biased)}
\]

\[
= \frac{9}{10} \cdot \frac{1}{2} + \frac{1}{10} = \frac{11}{20}
\]

12. In problem 11, your flip was H. What are the chances you picked a fair coin?

A 9/10.
B 9/11.
C 9/20.
D 11/20.
E None of the above

\[
\text{PL(fair|H)} = \frac{\text{PL(fair) \cdot PL(H)}}{\text{PL(H)}} = \frac{\frac{9}{10} \cdot \frac{1}{2}}{\frac{11}{20}} = \frac{9}{11}
\]

13. X is a uniform random variable taking a value in the set \(\{1, 2, 3, 4\}\). What is \(P[X = 2]\)?

A 0.1.
B 0.2.
C 0.3.
D 0.4.
E None of the above

All outcome probabilities the same.

4 outcomes \(\to\) each probability = \(\frac{1}{4}\)

\[
P[X = 2] = 0.25
\]

14. In Problem 13, what is \(P[X \geq 2]\)?

A 0.
B 0.25.
C 0.5.
D 0.75.
E None of the above

\[
P[X \geq 2] = P[X \in \{2, 3, 4\}] = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}
\]

\[
= 0.75
\]

15. In Problem 13, what is \(E[X]\)?

A 2.
B 2.5.
C 3.
D 3.5.
E None of the above

\[
E(X) = \sum_{x=1}^{4} x \cdot P(X)
\]

\[
= \sum_{x=1}^{4} x \cdot \frac{1}{4} = \frac{1}{4} \sum_{x=1}^{4} x
\]

\[
= \frac{10}{4}
\]

\[
= 2.5
\]
16. Roll a fair 6-sided die 4 times. What are the chances to roll exactly 2 fours?
   A 140/6^4.
   B 150/6^4.
   C 160/6^4.
   D 170/6^4.
   E None of the above.

   \[ n = 4 \quad P = \frac{1}{6} \quad P[X = 2] = \binom{4}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{6 \cdot 5^2}{6^4} = \frac{150}{6^4} \]

   Uniform Prob Space:
   \# outcomes: 6x6x6x6 = 6^4
   \# outcomes with 2 fours: \( \binom{4}{2} \) choices for where to place 4's.

   \[ \text{Product Rule: } \binom{4}{2} \cdot 5^2 \text{ choices for other 2 rolls} \]

   \[ \text{Prob } = \frac{150}{6^4} \]

17. A biased coin has probability of heads 3/4. What is the expected number of heads in 20 flips?
   A 5.
   B 10.
   C 15.
   D 20.
   E None of the above.

   \[ n = 20 \quad p = \frac{3}{4} \]

   \[ \mathbb{E}[\text{# successes}] = np = 20 \cdot \frac{3}{4} = 15 \]

18. A box has 5 fair and 5 two-headed coins. Pick a random coin and make 10 flips. What is \( \mathbb{E}[\text{number of heads}] \)?
   A 6
   B 6.5
   C 7
   D 7.5
   E None of the above.

   \[ \mathbb{E}[\text{# Heads}] = \mathbb{E}[\text{# Heads | Fair}] \cdot P(\text{Fair}) + \mathbb{E}[\text{# Heads | 2-Heads}] \cdot P(\text{2-Heads}) \]
   \[ = \left( \frac{10 \cdot \frac{1}{2}}{10} \cdot \frac{5}{10} \right) + \left( \frac{10 \cdot \frac{5}{2}}{10} \cdot \frac{5}{10} \right) \]
   \[ = \frac{5}{2} + 10 \cdot \frac{1}{2} \]
   \[ = 2.5 + 5 = 7.5 \]

19. Boys are twice as likely as girls. What is the expected number of kids till you have a boy?
   A 1.
   B 2.
   C 3.
   D 4.
   E None of the above.

   \[ P[B] = \frac{2}{3} \quad P[G] = \frac{1}{3} \quad \rightarrow P[G] = \frac{1}{3} \quad \rightarrow P[B] = \frac{2}{3} \]

   Expected Wait to Boy = \[ \frac{1}{P[B]} = \frac{3}{2} = 1.5 \]

20. Ana and Amy have kids till a boy. For Ana, boys and girls are equally likely. For Amy, boys are twice as likely as girls. What is the expected number of kids Ana and Amy will have in total?
   A 3.
   B 3.5.
   C 4.
   D 4.5.
   E None of the above.

   \[ X_1 = \# \text{ Kids for Ana} \quad \mathbb{E}[X_1] = \frac{1}{2} = 1 \]

   \[ X_2 = \# \text{ Kids for Amy} \quad \mathbb{E}[X_2] = \frac{1}{\text{ Prob of Boy}} = \frac{1}{2} = 1.5 \]

   Total = \( X_1 + X_2 \)

   \[ \mathbb{E}[X_1 + X_2] = \mathbb{E}[X_1] + \mathbb{E}[X_2] = 2 + 1.5 = 3.5 \]