

Quiz 2

60 Minutes

First Name: _____

Last Name: _____

RIN: _____

NO COLLABORATION or electronic devices.

Any violations will result in an F.

No questions allowed during the test unless you think there is a mistake.

GOOD LUCK!

Circle at most one answer per question.

10 points for each correct answer.

For [**Show Work**] problems, you **MUST** show correct work to get credit.

Correct answers with no explanation will get a 0.

Final Score: _____ / 200

1. When we grade FOCS quizzes, we order 3 pizzas. If the pizzeria has 10 types of pizza on the menu, how many different orders can we make (for example, {two pepperoni and one margherita} is the same order as {one margherita and two pepperoni})?
- A $\binom{10}{2}$
- B $10 \times 9 \times 8$
- C $\binom{10}{8}$
- D $10!$
- E None of the above.
2. **[Show Work]** Suppose you are ordering a build-your-own pizza, where you have to choose a crust type, a base type and two different toppings. You have a choice of two types of crust, two types of base and 10 toppings. How many build-your-own pizzas are possible?
- A $\binom{10}{2}$
- B 45
- C 90
- D 180
- E None of the above.
3. Suppose FOCS has 100 students. If I order students randomly, how many top 5 orderings are possible (assuming all grades are different)?
- A $100!$
- B $\binom{100}{5}$
- C $100 \times 99 \times 98 \times 97 \times 96$
- D 200
- E None of the above.
4. Let X and Y be random variables, and let Ω be the set of all outcomes. When is the following true: $\sum_{x \in X(\Omega)} \mathbb{P}[X = x] = \sum_{y \in Y(\Omega)} \mathbb{P}[Y = y]$?
- A Never.
- B Always.
- C Only when X and Y are the same variable.
- D There is not enough information to determine.
- E None of the above.
5. Suppose $X(\omega_1) = 1, X(\omega_2) = 1, X(\omega_3) = 0$. If $\Omega = \{\omega_1, \omega_2, \omega_3\}$, what is $X(\Omega)$?
- A $\{1\}$ B $\{1,1,0\}$ C $\{0,1\}$ D $\{\omega_1, \omega_2, \omega_3\}$ E None of the above.

6. Suppose A and B are events such that $A \subseteq B$. What do we know?

- A $\mathbb{P}[A] \leq \mathbb{P}[B]$
- B $\mathbb{P}[B] \leq \mathbb{P}[A]$
- C We don't have enough information to compare the probabilities of the events.
- D $\mathbb{P}[B] = 1$
- E None of the above.

7. [Show Work] Suppose X_1 and X_2 are independent and uniform on $\{1, 2, 3, 4, 5\}$. What is $\mathbb{P}[X_1 + X_2 > 3]$?

- A $\frac{21}{25}$
- B $\frac{22}{25}$
- C $\frac{23}{25}$
- D $\frac{24}{25}$
- E $\frac{25}{25}$

8. Suppose X_1 is the number of heads in n fair coin tosses, and X_2 is 1 if all coins match and 0, otherwise. What do we know about $\mathbb{E}[X_1 + X_2]$?

- A The variables are dependent, so the expected value is unknown.
- B $\mathbb{E}[X_1 + X_2] = \mathbb{E}[X_1] \times \mathbb{E}[X_2]$
- C $\mathbb{E}[X_1 + X_2] = 0$
- D $\mathbb{E}[X_1 + X_2] = \mathbb{E}[X_1] + \mathbb{E}[X_2]$
- E None of the above.

9. [Show Work] What is $\mathbb{E}[X_2]$ for the random variable X_2 defined in the previous question?

- A 0
- B $\frac{1}{2^n}$
- C $\frac{2}{2^n}$
- D $\frac{1}{2}$
- E 1

10. Suppose I randomly choose 6 different numbers from the set $\{1, \dots, 10\}$. What is the probability that my set of numbers contains at least two subsets that sum up to the same number?

- A $\frac{1}{\binom{10}{5}}$
- B $1 - \frac{1}{\binom{10}{5}}$
- C 0
- D 1
- E None of the above.

11. Suppose the answer to the first 5 questions is A, and the answer to the remaining 15 questions is chosen uniformly at random between A-E? How many A answers do you expect to see?

A 5
 B 6
 C 7
 D 8
 E 9

12. Suppose the correct answers to all 20 questions are chosen uniformly at random. What is the probability that a letter is never used?

A $\frac{\binom{20}{4}}{\binom{20}{5}}$
 B $5 \times \frac{\binom{20}{4}}{\binom{20}{5}}$
 C $\frac{4^{20}}{5^{20}}$
 D $5 \times \frac{4^{20}}{5^{20}}$
 E None of the above.

13. **[Show Work]** Suppose submittity only allows 1 student to submit at a given time – if multiple students try at the same time, no one succeeds. Suppose all 100 students start at the same time: each student flips a fair coin and if they get a H, they try to submit; otherwise, they wait 1s and try again. How much time do we expect to wait until the first successful submission?

A $\frac{1}{2}$ s
 B 2s
 C $\frac{2^{100}}{100}$ s
 D $\frac{100}{2^{100}}$ s
 E None of the above.

14. In Question 13, how much time do we expect to wait until the 3rd successful submission?

A $\frac{3}{2}$ s
 B 6s
 C $\frac{3 \times 2^{100}}{100}$ s
 D $(\frac{2^{100}}{100} + \frac{2^{99}}{99} + \frac{2^{98}}{98})$ s
 E None of the above.

15. **[Show Work]** You flip a fair coin until you get 2 consecutive H. But if you get a T before you get 2 H, you restart. How many flips do you expect to make?

A 4 B 5 C 6 D 7 E None of the above

16. Suppose the weather forecast is correct 95% of the time and it snows on 50% of days. What is the chance of snow if the forecast says it will not snow?

- A 2.5%
- B 5%
- C 10%
- D 20%
- E None of the above.

17. [Show Work] Consider the joint PDF of variables X and Y below. What is $\mathbb{E}[XY]$?

- A 2
- B 1
- C $\frac{1}{2}$
- D $\frac{5}{12}$
- E None of the above.

$P_{XY}(x, y)$		X			
		0	1	2	3
Y	0	$\frac{1}{4}$	0	$\frac{1}{8}$	$\frac{1}{8}$
	1	$\frac{1}{4}$	$\frac{1}{6}$	0	$\frac{1}{12}$

18. [Show Work] Consider the joint PDF of variables X and Y in Question 17. What is $\mathbb{E}[X]$?

- A $\frac{15}{24}$
- B $\frac{25}{24}$
- C 1
- D 2
- E None of the above.

19. Consider the joint PDF of variables X and Y in Question 17. How do we show that X and Y are not independent?

- A Show that $\mathbb{P}[X = x, Y = y] = \mathbb{P}[X = x]\mathbb{P}[Y = y]$ for some x, y .
- B Show that $\mathbb{P}[X = x, Y = y] = \mathbb{P}[X = x]\mathbb{P}[Y = y]$ for all x, y .
- C Show that $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$.
- D They are independent.
- E None of the above.

20. X and Y are independent Binomial random variables over n trials with success probabilities p_1 and p_2 , respectively. What is $\mathbb{E}[XY]$?

- A n
- B np_1p_2
- C $n^2p_1p_2$
- D $n(p_1 + p_2)$
- E None of the above

Scratch