

## Quiz 3

60 Minutes

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

RIN: \_\_\_\_\_

**NO COLLABORATION or electronic devices.**

**Any violations will result in an F.**

**No questions** allowed during the test unless you think there is a mistake.

**GOOD LUCK!**

Circle at most one answer per question.

**10 points** for each correct answer.

For [**Show Work**] problems, you **MUST** show correct work to get credit.

Correct answers with no explanation will get a 0.

Final Score: \_\_\_\_\_ / 200

1. Consider the sets  $A = \{1, 2, 3\}$  and  $B = \{4, 5\}$ . Which function is an injection from  $A$  to  $B$ ?

A  $f(1) = 4, f(2) = 4, f(3) = 5$

B  $f(1) = 4, f(2) = 5, f(3) = 5$

C  $f(1) = 4, f(2) = 4$

D  $f(1) = 4, f(3) = 5$

E None of the above.

2. Consider the sets  $A = \{1, 2, 3\}$  and  $B = \{4, 5\}$ . Which function is a surjection from  $A$  to  $B$ ?

A  $f(1) = 4, f(2) = 4, f(3) = 5$

B  $f(1) = 5, f(2) = 5, f(3) = 5$

C  $f(1) = 4, f(2) = 4$

D  $f(1) = 5, f(3) = 5$

E None of the above.

3. Consider the real intervals  $A = [0, 1]$ ,  $B = [-1, 1]$ . Which function is a bijection from  $A$  to  $B$ ?

A  $f(x) = 2x + 2$

B  $f(x) = 2x + 1$

C  $f(x) = 2x - 1$

D  $f(x) = 2x$

E None of the above.

4. Consider the real interval  $A = [0, 1]$  and the set  $B$  of all *rational*s in the interval  $[-1, 1]$ . Which function is a bijection from  $A$  to  $B$ ?

A  $f(x) = 2x + 2$

B  $f(x) = 2x + 1$

C  $f(x) = 2x - 1$

D  $f(x) = 2x$

E None of the above.

5. Which of the following sets is countable (recall  $\mathbb{N}$  is natural numbers,  $\mathbb{Q}$  is rationals)?

A The set of all functions from  $\mathbb{N}$  to  $\mathbb{N}$

B The set of all functions from  $\mathbb{N}$  to  $\mathbb{Q}$

C The set of all infinite binary strings

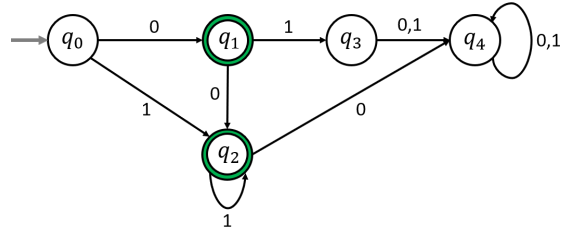
D The set of all pairs of natural numbers  $(m, n)$ , where  $m, n \in \mathbb{N}$

E None of the above

6. How do we know the set of all functions is uncountable?
- A The set of all programs is uncountable.
  - B There is a bijection from the set of all binary functions defined on  $\mathbb{N}$  to the set of all infinite binary strings.
  - C The set of finite binary strings is uncountable.
  - D The set of all functions is countable.
  - E None of the above.
7. [Show Work] How many surjections from  $\{1, 2, 3, 4, 5\}$  to  $\{1, 2\}$  are there?
- A 10
  - B 20
  - C 30
  - D 32
  - E None of the above.
8. What is the relationship between regular languages and DFAs?
- A There exist regular languages which cannot be decided by a DFA.
  - B There exist languages which can be decided by a DFA but cannot be described by a regular expression.
  - C Regular languages is the set of languages that can be decided by DFAs.
  - D All languages are regular.
  - E None of the above.
9. Consider the language  $\mathcal{L} = \{1, 01\}^*$ . Which string is not in  $\mathcal{L}$ ?
- A 11
  - B 011
  - C 010101
  - D 10101011
  - E They are all in  $\mathcal{L}$ .
10. Consider the language  $\mathcal{L} = \{0^{n-1}1^{3n} \mid n \geq 0\}^*$ . What do we know about  $\mathcal{L}$ ?
- A  $\mathcal{L}$  contains all strings with three times as many 1s as 0s.
  - B  $\mathcal{L}$  contains all strings that start with a 0.
  - C  $\mathcal{L}$  is regular.
  - D  $\mathcal{L}$  is finite.
  - E None of the above.

11. Consider the deterministic finite automaton (DFA) below. What is the language decided by this machine?

- A  $\mathcal{L} = \{0, 00, 1\}$   
 B  $\mathcal{L} = \{0, 00, 1\}^*$   
 C  $\mathcal{L} = \{0, 00\} \cup \{1\}^*$   
 D  $\mathcal{L} = \{\varepsilon, 0, 00\} \cup \{1\}^*$   
 E None of the above.



12. Consider the DFA in Question 11. Which state does the DFA terminate in for the input string 0010?

- A  $q_1$   
 B  $q_2$   
 C  $q_3$   
 D  $q_4$   
 E None of the above

13. Consider the language  $\mathcal{L} = \{01\}^* \{0\}^* \{10\}^*$ . Which word is not in  $\mathcal{L}$ ?

- A 0110  
 B 011  
 C 010  
 D  $\varepsilon$   
 E They are all in  $\mathcal{L}$ .

14. [Show Work] Which of the following languages cannot be solved by a DFA?

- A  $\mathcal{L} = \{\text{strings with at least three 1s and at least three 0s}\}$   
 B  $\mathcal{L} = \{\text{strings with at least five million 0s}\}$   
 C  $\mathcal{L} = \{\text{strings with no 1s and no 0s}\}$   
 D  $\mathcal{L} = \{\text{strings of even length}\}$   
 E They are all regular languages.

15. Which of the following strings *cannot* be generated by the CFG:  $S \rightarrow \varepsilon \mid 1S \mid 0S$ ?

- A 1001  
 B 0011  
 C 1100  
 D 1010  
 E They can all be generated.

16. [Show Work] Which CFG generates the language of all strings of odd length?
- A  $S \rightarrow \varepsilon \mid 1S0 \mid 0S1$
  - B  $S \rightarrow \varepsilon \mid 1S \mid 0S$
  - C  $S \rightarrow 0 \mid 1 \mid 1S0 \mid 0S1 \mid 0S0 \mid 1S1$
  - D  $S \rightarrow 0 \mid 1 \mid 1S0S \mid S0S1$
  - E None of the above.
17. Which of the following regular expressions *cannot* be generated by a CFG?
- A  $1 * 0$
  - B  $(01 * 1 * 0)^*$
  - C  $\overline{\{0, 1\}}$
  - D  $\{0, 1\}^*$
  - E All regular expressions can be generated by a CFG.
18. What is the relationship pushdown automata (PDAs) and Turing Machines (TMs)?
- A They can decide the same set of languages.
  - B TMs can decide some languages that no PDA can decide.
  - C PDAs can decide some languages that no TM can decide.
  - D They are the same model.
  - E None of the above.
19. Each TM description can be written as a finite binary string. What do we know?
- A The number of TMs is uncountable.
  - B The number of TMs is the same as the number of infinite binary strings.
  - C The number of TMs is countable.
  - D There is a bijection between the set of TMs and the set of infinite binary strings.
  - E None of the above.
20. Which statement is true about the language  $\mathcal{L} = \{w\#w^R \mid w \in \{0, 1\}^*\}$ ?
- A A DFA can solve this language. A TM can solve this language.
  - B A DFA cannot solve this language. A PDA can solve this language.
  - C A PDA can solve this language. A TM cannot solve this language.
  - D A PDA cannot solve this language. A TM cannot solve this language.
  - E None of the above.

Scratch